Mathematics 3200
More Counting

Name: Key

1. The Fundamental Counting principal is a means of finding the number of ways of performing two or more operations together.

a. Jill is trying to select a new cell phone based on the following:

Brands: Ace, Best, Cutest (3)

Colour: Lime, Magenta, Navy, Orange (4)

Plans: Text, Unlimited Calling (2)

Determine the number of ways of selecting her new cell phone.

$3 \times 4 \times 2 = 24$

b. In Newfoundland and Labrador, a license plate consists of a letter-letter-letter-digit-digit-digit arrangement such as CXT 132.

(a) How many license plates are possible? $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17{,}576{,}000$

(b) How many license plates are possible if no letter or digit can be repeated? $26 \times 25 \times 24 \times 10 \times 9 \times 8 = 11{,}232{,}000$

(c) How many license plates are possible if vowels (a, e, i, o, u) are not allowed? (repetition is allowed)

$21 \times 21 \times 21 \times 10 \times 10 \times 10 = 9{,}261{,}000$

2. A permutation is an ordered arrangement of all or part of a set. For example, the possible permutations of the letters A, B and C are ABC, ACB, BAC, BCA, CAB and CBA. The order of the letters matters.

$${}_nP_r = \frac{n!}{(n-r)!}$$

a. How many two-digit numbers can be formed using the digits 1, 2, 3, 4, 5, 6 if repetition is allowed?

$6 \times 6 = 36$ (can't use formula if repetition is allowed)

b. The code for a lock consists of four numbers selected from 0, 1, 2, 3 with no repeats. For example, the code 1-2-1-3 would not be allowed but 3-0-2-1 would be allowed. Using the permutation formula, determine the number of possible codes.

$\underline{4} \times \underline{3} \times \underline{2}\ \underline{1} = 24$

or

$${}_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4 \times 3 \times 2 \times 1}{1} = 24 \text{ possible codes}$$

c. $_{n+1}P_2 = 20$

$\frac{(n+1)!}{(n+1-2)!} = 20$

$\frac{(n+1)!}{(n-1)!} = 20$

$\frac{(n+1)(n)(n-1)!}{(n-1)!} = 20$

$n^2+n-20=0$

$(n+5)(n-4)=0$

$n \neq -5$, $\underline{\underline{n=4}}$

d. Denise has a set of posters to arrange on her bedroom wall. She can only fit two posters side by side. If there are 72 ways to choose and arrange two posters, how many posters does she have in total?

$_nP_2 = 72$

$\frac{n!}{(n-2)!} = 72$

$\frac{(n)(n-1)(n-2)!}{(n-2)!} = 72$

$n^2-n-72=0$

$(n-9)(n+8)=0$

$\underline{\underline{n=9}}$ $\}$ $n \neq -8$

3. Combinations:

In contrast to permutations, combinations are an arrangement of objects without regard for order. A formula will be developed and applied in problem solving situations.

Generally, given a set of n objects taken r at a time, the number of possible combinations is $_nC_r = \frac{n!}{(n-r)!r!}$.

Note that the notation $\binom{n}{r}$ is sometimes used instead of $_nC_r$.

Which situations below would be a permutation/combination or FCP?

- Adam, Marie and Brian are standing in a line at a banking machine. In how many ways could they order themselves? → $3!$

order NOT imp → Paul, Renee and Emily are members of a committee. In how many ways could two of them be selected to attend a conference? → $_3C_2 = \frac{3!}{1!2!} = 3$

→ How many ways can a committee of three people be selected from a group of 12 people? → $_{12}C_4 = 495$

NOT imp

How many ways can three of eight people line up at a ticket counter? → $_8P_3 = 336$

imp

How many four-digit numbers are there?

$\underline{9}\ \underline{10}\ \underline{10}\ \underline{10} = 9000$

CANT be 0.

Evaluate: a. $_6C_2$ $= \frac{6!}{4!2!} = 15$

b. $_{12}C_{10}$ $\frac{12!}{2!10!} = 66$

c. $\binom{7}{3} = \frac{7!}{4!3!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$