Calc. of Trig.

1\. a) $y = \frac{\sin x}{x^2} \rightarrow y' = \frac{x^2(\cos x) - 2x \cdot \sin x}{x^4} \rightarrow \boxed{y' = \frac{x\cos x - 2\sin x}{x^3}}$

b) $\boxed{y' = 3\cos x - \frac{1}{2}\sin\left(\frac{x}{2}\right)}$

c) $y' = \cos\left(\sqrt[3]{2x+1}\right) \cdot \frac{1}{3}(2x+1)^{-2/3} \cdot 2$

$\boxed{y' = \frac{2\cos\sqrt[3]{2x+1}}{3\sqrt[3]{(2x+1)^2}}}$

d) $y' = 7\sin^6\sqrt{x} \cdot$

$\boxed{y' = 7\sin^6\sqrt{x} \cdot \cos\sqrt{x} \cdot \frac{1}{2}x^{-1/2}} \rightarrow \boxed{y' = \frac{7\sin^6\sqrt{x}\cos\sqrt{x}}{2\sqrt{x}}}$

e) $y' = \cos x \cdot \tan x \cdot (-\csc x \cot x) + \cos x \sec^2 x \csc x + -\sin x \tan x \csc x$

$\boxed{y' = -\cot x + \frac{1}{\cos x \sin x} - \tan x}$ $\quad y' = \frac{-1}{\tan x} + \frac{1}{\sin x \cos x} - \frac{\tan^2 x}{\tan x}$

$y' = \frac{-(1+\tan^2 x)}{\tan x} + \frac{1}{\sin x \cos}$

$y' = \frac{-\sec^2 x}{\tan x} + \frac{1}{\sin x \cos}$

$y' = \frac{-\sec^2 x \cdot \cos x}{\sin x} + \frac{1}{\sin x \cos}$

$y' = \frac{-\sec^2 x \cos^2 x + 1}{\sin x \cos x}$

e) $\boxed{y' = 0}$

f) $y' = \frac{\cos x(x\cos x + 1\sin x) + \sin x(x\sin x)}{\cos^2 x}$

$y' = \frac{x\cos^2 x + \sin x\cos x + x\sin^2 x}{\cos^2 x}$

$y' = \frac{x(\cos^2 x + \sin^2 x) + \sin x\cos x}{\cos^2 x}$

$\boxed{y' = \frac{x + \sin x \cos x}{\cos^2 x}}$

g) $\boxed{y' = (x-5)^8 \cdot 7\csc^2(7x) + 8(x-5)^7\cot(7x)}$

h) $y' = 4\tan^3\sqrt{1+3x} \cdot \sec^2\sqrt{1+3x} \cdot \frac{1}{2}(1+3x)^{-1/2} \cdot 3$

$\boxed{y' = \frac{6\tan^3\sqrt{1+3x} \cdot \sec^2\sqrt{1+3x}}{\sqrt{1+3x}}}$

i) $y' = 15\cos^2(1+\tan x) \cdot -\sin(1+\tan x) \cdot (\sec^2 x)$

$\boxed{y' = -15\cos^2(1+\tan x) \cdot \sin(1+\tan x) \cdot \sec^2 x}$

j) $y' = \dfrac{(1+\sin x)(-\sin x) - (\cos x)\cdot\cos x}{(1+\sin x)^2}$

$y' = \dfrac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} \rightarrow y' = \dfrac{-\sin x - 1(\sin^2 x + \cos^2 x)}{(1+\sin x)^2}$

$\boxed{y' = \dfrac{-\sin x - 1}{(1+\sin x)^2}}$

k) $y' = 2\sec x \cdot \sec x \tan x + \sec(x^2)\tan(x^2)\cdot 2x$

$\boxed{y' = 2\sec^2 x \tan x + 2x\sec(x^2)\tan(x^2)}$

l) $y' = \sec^2(x^{-3}) \cdot -3x^{-4} \rightarrow \boxed{y' = \dfrac{-3\sec^2(x^{-3})}{x^4}}$

m) $y' = 2\tan(\pi - x^3)\cdot \sec^2(\pi - x^3)\cdot -3x^2$

$\boxed{y' = -6x^2\tan(\pi - x^3)\sec^2(\pi - x^3)}$

n) $y' = \dfrac{(1-x)(2\sec^2(3x)\cdot 3) + (1 + 2\tan(3x))}{(1-x)^2}$

$\boxed{y' = \dfrac{6\sec^2(3x) - 6x\sec^2(3x) + 1 + 2\tan(3x)}{(1-x)^2}}$

o) $y' = \frac{1}{2}(1+\sin 2x)^{-1/2}\cdot 2\cos 2x \rightarrow \boxed{y' = \dfrac{\cos 2x}{\sqrt{1+\sin 2x}}}$

p) $y' = \dfrac{1}{1+(3x)^2}\cdot 3 \rightarrow \boxed{y' = \dfrac{3}{1+9x^2}}$

q) $y' = \cos(\cos^{-1}x)\cdot \dfrac{-1}{\sqrt{1-x^2}} \rightarrow \boxed{y' = \dfrac{-x}{\sqrt{1-x^2}}}$

r) $y' = \dfrac{-1}{\sqrt{1-(2x-1)}}\cdot \frac{1}{2}(2x-1)^{-1/2}\cdot 2$

$\boxed{y' = \dfrac{-1}{\sqrt{2x-1}\cdot\sqrt{2-2x}}}$

2. $f' = -\sin 3x \cdot 3 \rightarrow f' = -3\sin 3x$

$f'' = -9\cos 3x$

$f''' = 27\sin 3x$

$\boxed{f^4 = 81\cos 3x}$

#3 (omit for now)

#4 a) $y^2 \cdot 2\cos x \cdot -\sin x + 2y \cdot y' \cdot \cos^2(x) = 0$

$2yy'\cos^2 x = 2y^2 \sin x \cos x$

$y' = \frac{\cancel{2y^2}^{\,y} \sin x \cancel{\cos x}}{\cancel{2y} \cancel{\cos^2 x}_{\cos x}} \rightarrow y' = \frac{y\sin x}{\cos x}$

$\boxed{y' = y\tan x}$

b) $1 + \cos y \cdot y' = xy' + 1y$

$y'\cos y - xy' = y - 1$

$y'(\cos y - x) = y - 1$

$\boxed{y' = \frac{y-1}{\cos y - x}}$

c) $x^2 \cdot -\sin y \cdot y' + 2x\cos y = y^2 \cdot \cos x + 2y \cdot y' \sin x$

$-y'x^2 \sin y - 2y \cdot y' \sin x = y^2 \cos x - 2x\cos y$

$y'(-x^2 \sin y - 2y\sin x) = y^2 \cos x - 2x\cos y$

$\boxed{y' = \frac{y^2\cos x - 2x\cos y}{-x^2\sin y - 2y\sin x}}$

#5. $\pi(2x - 2yy') = -\sin(\pi y) \cdot \pi \cdot y'$

$2\pi x - 2\pi yy' + \pi y' \sin \pi y = 0$

$y'(-2\pi y + \pi \sin \pi y) = -2\pi x$

$y' = \frac{-2\pi x}{-2\pi y + \pi \sin \pi y}$

$y' = \frac{2x}{2y - \sin \pi y}$

$y'\Big|_{\frac{3}{2}, \frac{3}{2}} = \frac{2\left(\frac{3}{2}\right)}{2\left(\frac{3}{2}\right) - \sin\frac{3\pi}{2}} = \frac{3}{3 - -1} = \boxed{\frac{3}{4}}$

final ans:

$\boxed{y - \frac{3}{2} = \frac{3}{4}\left(x - \frac{3}{2}\right)}$

#6

$y = \sin x \tan x$

$y' = \sin x \cdot \sec^2 x + \cos x \cdot \tan x$

$y' = \dfrac{\sin x}{\cos^2 x} + \dfrac{\sin x}{1}$

$y'\Big|_{\frac{\pi}{6}} = \dfrac{\frac{1}{2}}{\frac{3}{4}} + \frac{1}{2}$

$\frac{1}{}$

$= \frac{2}{3} + \frac{1}{2}$

$= \boxed{\frac{7}{6}}$

ordered pr

$y = \sin\frac{\pi}{6} \cdot \tan\frac{\pi}{6}$

$= \frac{1}{2} \cdot \dfrac{\frac{1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}}$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{3}}$

$= \frac{1}{2\sqrt{3}}$

$\left(\frac{\pi}{6}, \frac{1}{2\sqrt{3}}\right)$

Tangent line $\Rightarrow$ $y - \frac{1}{2\sqrt{3}} = \frac{7}{6}\left(x - \frac{\pi}{6}\right)$

Normal line $\Rightarrow$ $y - \frac{1}{2\sqrt{3}} = -\frac{6}{7}\left(x - \frac{\pi}{6}\right)$

#7.

tangent line $\rightarrow$ $y - 0 = m\left(x - \frac{\pi}{6}\right)$

$y' = -\sin 3x \cdot 3$

$y'\Big|_{\frac{\pi}{6}} = -3 \cdot \sin\frac{\pi}{2} = -3 \cdot 1 = \boxed{-3}$

tangent line $\rightarrow$ $y = -3\left(x - \frac{\pi}{6}\right)$

tangent line would be horizontal if y' were = to 0

so $0 = -3\sin 3x$

$0 = \sin 3x$

1st possibility $\sin^{-1}(0) = 3x$

$0 = 3x$

$\boxed{0 = x}$

2nd possibility

$3x = \pi$

$\boxed{x = \frac{\pi}{3}}$

3rd possibility

$3x = 2\pi$

$\boxed{x = \frac{2\pi}{3}}$

4th possibility

$3x = 4\pi$

$\boxed{x = \frac{4\pi}{3}}$

5th solution

$3x = 5\pi$

$\boxed{x = \frac{5\pi}{3}}$

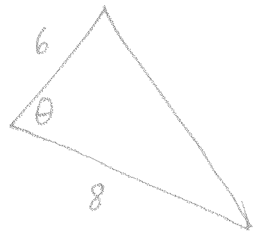
6 solution

$3x = 6\pi$

$\boxed{x = 2\pi}$

stop here

9.



$\frac{d\theta}{dt} = -0.035$ rad/s

find $\frac{dA}{dt}$ when $\theta = \frac{\pi}{6}$

$A = \frac{1}{2} \cdot 6 \cdot 8 \cdot \sin\theta$

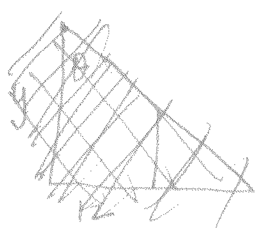
$A = 24 \sin\theta$

$\frac{dA}{dt} = 24 \cdot \cos\theta \cdot \frac{d\theta}{dt}$

$= 24 \cos\frac{\pi}{6} \cdot (-0.035)$

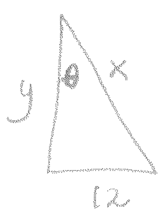
$= \boxed{-.727 \ m^2/s}$

8.



$\frac{dy}{dt} = 10$ m/min

I think there's a problem with this question



$\frac{dy}{dt} = 10$ m/min

find : $\frac{d\theta}{dt}$ when $x = 24$

~~cos θ =~~

$\sin\theta = \frac{12}{24}$

$\theta = \frac{\pi}{6}$

$\cos\theta = \frac{y}{x}$

$x \cos\theta = y$

$x \cdot -\sin\theta \cdot \frac{d\theta}{dt} + 1 \cos\theta = \frac{dy}{dt}$

$24 \cdot -\sin\frac{\pi}{6} \cdot \frac{d\theta}{dt} + 1 \cdot \cos\frac{\pi}{6} = 10$

$24\left(-\frac{1}{2}\right)\frac{d\theta}{dt} + \frac{\sqrt{3}}{2} = 10$

$-12 \frac{d\theta}{dt} = 10 - .866$

$\frac{d\theta}{dt} = \frac{9.134}{-12}$

$\frac{d\theta}{dt} = -.761$ rad/min