Calculus 3208 **Unit: The Calculus of Trigonometry**

1. Determine the derivative of each:

a. $f(x) = \dfrac{\sin x}{x^2}$

$$f'(x) = \frac{\cos x\,(x^2) - 2x(\sin x)}{(x^2)^2}$$

$$= \frac{x\left[x \cos x - 2 \sin x\right]}{x^4}$$

$$= \frac{x \cos x - 2 \sin x}{x^3}$$

b. $f(x) = \dfrac{1}{2}\sin^2(5x) + \tan x - 1$

$$f'(x) = \tfrac{1}{2}(2 \sin 5x)\cdot \cos(5x)(5) + \sec^2 x - 0$$

$$= 5 \sin(5x) \cos(5x) + \sec^2 x$$

c. $f(x) = \sec x \csc x$

$$f'(x) = (\sec x \tan x)(\csc x) + (-\csc x \cot x)(\sec x)$$

$$f'(x) = \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}\right) - \left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x}\right)$$

$$= \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}$$

$$= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \cdot \sin^2 x}$$

d. $f(x) = \tan \sqrt[3]{6x+1}$

$$f'(x) = \sec^2 \sqrt[3]{6x+1} \cdot \tfrac{1}{3}(6x+1)^{-\frac{2}{3}} \cdot 6$$

$$= \frac{2 \sec^2 \sqrt[3]{6x+1}}{\left(\sqrt[3]{6x+1}\right)^2}$$

2. Determine $\dfrac{dy}{dx}$ given: $x\cos(xy) = y\sin(3x)$

$$(1)\cos(xy) + -x\sin(xy)\cdot\left[y + x\frac{dy}{dx}\right] = \frac{dy}{dx}\sin(3x) + 3y(\cos 3x)$$

$$\cos(xy) - xy\sin(xy) - x^2\sin(xy)\frac{dy}{dx} = \frac{dy}{dx}\sin(3x) + 3y\cos(3x)$$

$$\cos(xy) - xy\sin(xy) - 3y\cos(3x) = \sin(3x)\frac{dy}{dx} + x^2\sin(xy)\frac{dy}{dx}$$

$$\frac{\cos(xy) - xy\sin(xy) - 3y\cos(3x)}{\sin(3x) + x^2\sin(xy)} = \frac{dy}{dx}$$

$$\frac{\cos\left(\frac{3\pi}{3}\right)}{\sin\left(\frac{2\pi}{3}\right)} = \frac{-1}{\sqrt{3}/2} = -\frac{2}{\sqrt{3}}$$

3. Determine the equation of the tangent line to the curve defined by the equation:

$y = \dfrac{\cos(3x)}{\sin(2x)}$ at $x = \dfrac{\pi}{3}$

$$\frac{dy}{dx} = \frac{-3\sin(3x)\cdot\sin 2x - 2\cos(2x)\cos(3x)}{\sin^2(2x)}$$

$$= \frac{-3\sin\left(\frac{3\pi}{3}\right)\cdot\sin\left(\frac{2\pi}{3}\right) - 2\cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{3\pi}{3}\right)}{\sin^2\left(\frac{2\pi}{3}\right)}$$

$$= \frac{-3(0)\left(\frac{\sqrt{3}}{2}\right) - 2\left(\frac{-1}{2}\right)(-1)}{\left(\sqrt{3}/2\right)^2}$$

$$= \frac{-1}{3/4} \Rightarrow -4/3$$

$$\therefore \left(y - \frac{-2}{\sqrt{3}}\right) = \frac{-4}{3}\left(x - \frac{\pi}{3}\right)$$

4. Determine the equation of the tangent line to the curve defined by the equation:

$\pi(x^2 + y^2) = \cos(\pi y)$ at $\left(\dfrac{3}{2}, \dfrac{3}{2}\right)$

$$\pi x^2 + \pi y^2 = \cos(\pi y)$$

$$2\pi x + 2\pi y\frac{dy}{dx} = -\cos(\pi y)\,\pi\frac{dy}{dx}$$

$$2\pi y\frac{dy}{dx} + \pi\cos(\pi y)\frac{dy}{dx} = -2\pi x$$

$$\frac{dy}{dx} = \frac{-2\pi x}{2\pi y + \pi\cos\pi y}$$

AT $(3/2, 3/2)$

$$m = \frac{-2\pi(3/2)}{2\pi(3/2) + \pi\cos\left(\frac{3\pi}{2}\right)}$$

$$m = \frac{-3}{3+0} = -1$$

$$\therefore (y - 3/2) = -1(x - 3/2)$$

5. The position of a particle as it moves horizontally is described by the equation $s = 2\sin t - \cos t$, $0 \le t \le 2\pi$, where s is displacement in metres and t is the time in seconds. Find the absolute maximum and minimum displacements.

$$s' = 2\cos t - -\sin t$$

$$s' = 2\cos t + \sin t$$

$$s' = 0 \Rightarrow 2\cos t + \sin t = 0$$

$$\frac{2\cos t}{\cos t} = \frac{-\sin t}{\cos t}$$

$$-2 = \tan t$$

$$t = \tan^{-1}(2)$$

$t = 1.107$ (Ref ∠)

TAN IS Neg IN Q2 & Q4

Q2 $t = \pi - 1.107 = 2.07$ seconds

Q4 $t = 2\pi - 1.07 = 5.21$ seconds