Limits and Absloute Value.

Recall: $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ or $|x-a| = \begin{cases} x-a, & x \geq a \\ -(x-a), & x < a \end{cases}$

Evaluate each limit

1. $\lim\limits_{x \to 2^+} \frac{x^2-4}{|x-2|}$

$= \lim\limits_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$x-2=0$

$\boxed{x=2}$

$|x-2| = \begin{cases} x-2, & x \geq 2 \\ -(x-2), & x < 2 \end{cases}$

$= \lim\limits_{x \to 2^+} x+2$

$= 4$

2. $\lim\limits_{x \to 3^-} \frac{x^2+2x-\cancel{15}\ 15}{|x-3|}$

$= \lim\limits_{x \to 3^-} \frac{\cancel{(x-3)}(x+5)}{-\cancel{(x-3)}}$

$= \lim\limits_{x \to 3^-} \frac{x+5}{-1}$

$= \frac{8}{-1}$

$= -8$

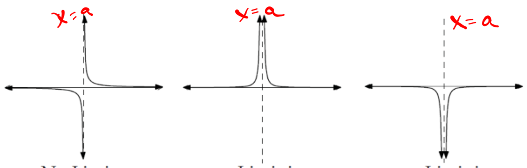
3. $\lim\limits_{x \to -1^+} \frac{x^3+1}{|x+1|}$

$= \lim\limits_{x \to -1^+} \frac{\cancel{(x+1)}(x^2-1x+1)}{\cancel{(x+1)}}$

$= \lim\limits_{x \to -1^+} x^2-1x+1$

$= 3$

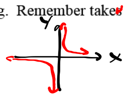
When when determining $\lim\limits_{x \to a} \frac{f(x)}{g(x)}$ and direct substitution yields $\frac{k}{0}$, where $k \neq 0$, this will yield one of the following situations.



No Limit   Limit is ∞   Limit is -∞

Consider the following. Remember take~~s~~ limit from both left and right.

a. $\lim\limits_{x \to 0} \frac{1}{x}$

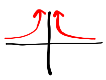


$\lim\limits_{x \to 0^+} \frac{1}{x} = \frac{+}{+} = +\infty$

$\lim\limits_{x \to 0^-} \frac{1}{x} = \frac{+}{-} = -\infty$

$\therefore \lim\limits_{x \to 0} \frac{1}{x}$ DNE

b. $\lim\limits_{x \to 0} \frac{1}{x^2}$



$\lim\limits_{x \to 0^+} \frac{1}{x^2} = \frac{+}{+} = +\infty$

$\lim\limits_{x \to 0^-} \frac{1}{x^2} = \frac{+}{+} = +\infty$

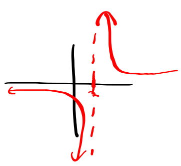
$\therefore \lim\limits_{x \to 0} \frac{1}{x^2} = +\infty$

c. $\lim\limits_{x \to 2} \frac{x+4}{x-2}$

$\lim\limits_{x \to 2^+} \frac{x+4}{x-2} = \frac{+}{+} = +\infty$

$\lim\limits_{x \to 2^-} \frac{x+4}{x-2} = \frac{+}{-} = -\infty$

$\therefore \lim\limits_{x \to 2} \frac{x+4}{x-2}$ dne



- as x approaches 2 from the left, the numerator approaches 6, and the denominator approaches 0 through negative values: $\lim_{x\to 2^-}\frac{x+4}{x-2}=-\infty$
- as x approaches 2 from the right, the numerator approaches 6, and the denominator approaches 0 through positive values: $\lim_{x\to 2^+}\frac{x+4}{x-2}=\infty$
- the $\lim_{x\to 2}\frac{x+4}{x-2}$ does not exist
- the function has a vertical asymptote at $x = 2$

You try:

1. $\lim_{x\to 3}\frac{x+1}{x-3}$
2. $\lim_{x\to -1}\frac{x}{(x+1)^2}$
3. $\lim_{x\to 5}\frac{3x}{x-5}$
4. $\lim_{x\to 0}\frac{x-5}{x^4}$