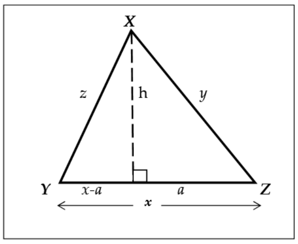
Section 2.4 – The Cosine Law

The Cosine Law relates the measure of one of the angles and all three sides of a triangle.



1. Using the right triangle on the right and the Cosine ratio, solve for a.

$\cos Z = \frac{a}{y} \rightarrow \boxed{y\cos Z = a}$

2. Using both right triangles, set up two Pythagorean equations:

$z^2 = h^2 + (x-a)^2$ $\qquad$ $y^2 = a^2 + h^2$

3. Expand the first Pythagorean equation:

$z^2 = h^2 + x^2 - 2ax + a^2$

$z^2 = x^2 + a^2 + h^2 - 2ax$

4. Notice that the last two terms of the expanded equation are the same as the second equation in step 2. Since the sum of these terms is equal to y^2, substitute y^2 for the terms. Also, substitute the a for the equation determined in step 1.

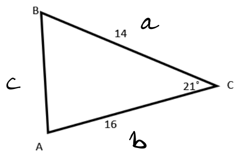
5. Rearrange the equation to get:

$z^2 = x^2 + y^2 - 2xa$

$\boxed{z^2 = x^2 + y^2 - 2xy\cos Z}$

$\frac{c^2 - a^2 - b^2}{-2ab}$

Example 1: Find the measure of side c.



$c^2 = a^2 + b^2 - 2ab\cos C$

(why did we set up the equation in this way?)

$c^2 = 14^2 + 16^2 - 2(14)(16)(\cos 21)$

$c^2 = 33.76$

$c = \sqrt{33.76}$

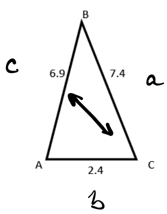
$c = 5.8$

To find a missing angle if you know the measures of all three sides, rearrange the formula to solve for cosZ.

$$\cos Z = \frac{x^2 + y^2 - z^2}{2xy}$$

Example 2: Find the measure of angle C

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

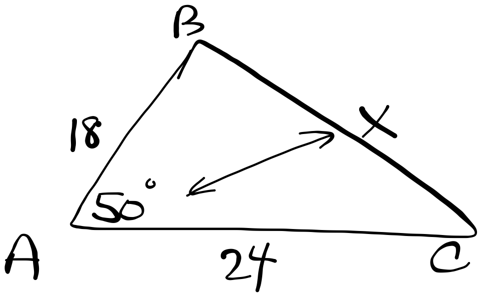


$\cos C = \frac{7.4^2 + 2.4^2 - 6.9^2}{2(7.4)(2.4)}$

$\cos C = \frac{12.91}{35.5}$

$C = \cos^{-1}\left(\frac{12.91}{35.5}\right)$

$C = 68.7^\circ$



$$a^2 = b^2 + c^2 - 2(b)(c)(\cos A)$$

$$x^2 = 18^2 + 24^2 - 2(18)(24)(\cos 50)$$

$$x^2 = 344.6$$

$$x = \sqrt{344.6}$$

$$x = 18.6$$

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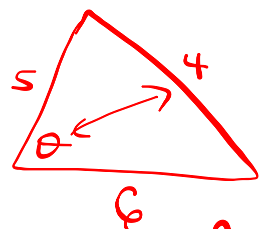
#3 A Triangle has sides measuring 4, 5, 6. What is the measure of the smallest Angle?

Soln



* Smallest Angle Across from smallest Side

$$\cos\theta = \frac{5^2 + 6^2 - 4^2}{2(5)(6)}$$

$$\cos\theta = \frac{45}{60}$$

$$\theta = \cos^{-1}\left(\frac{45}{60}\right)$$

$$\theta = 41^\circ$$

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