

Midterm Examination
Math 1201
January 2012

Value: 60 Marks

Time: 2 Hours

General Instructions

1. Candidates are required to do ALL items.
2. The examination consists of the following parts:
PART I: Multiple Choice (30 Questions) 30 Marks
Part II: Short Answer (3 Questions) 9 Marks
PART III: Long Answer (5 Questions) 21 Marks
3. A graphing calculator may be used for calculations and to obtain special values.
4. Answers to the **PART I** multiple choice questions are to be placed on the sheet provided. Only that sheet and the answers to the **PART II** and **Part III** items should be scanned and emailed to the teacher.
5. For **PART II** and **Part III** items, candidates are reminded to show all necessary steps and calculations as credit may be given for incomplete or partially correct solutions. Correct answers without calculations will not merit full marks.

****Check to ensure there are no missing pages****

Formulae Sheet

1. Sphere

$$\text{Surface Area: } S.A. = 4\pi r^2$$

$$\text{Volume: } V = \frac{4}{3}\pi r^3$$

2. Cone

$$\text{Surface Area: } S.A. = \pi rs + \pi r^2$$

3. Cylinder

$$S.A. = 2\pi rh + 2\pi r^2$$

4.

SI Units to Imperial Units Imperial Units to SI Units

$$1 \text{ mm} \doteq \frac{4}{100} \text{ in.}$$

$$1 \text{ in.} \doteq 2.5 \text{ cm}$$

$$1 \text{ cm} \doteq \frac{4}{10} \text{ in.}$$

$$1 \text{ ft.} \doteq 30 \text{ cm}$$

$$1 \text{ ft.} \doteq 0.3 \text{ m}$$

$$1 \text{ m} \doteq 39 \text{ in.}$$

$$1 \text{ yd.} \doteq 90 \text{ cm}$$

$$1 \text{ m} \doteq 3 \frac{1}{4} \text{ ft.}$$

$$1 \text{ yd.} \doteq 0.9 \text{ m}$$

$$1 \text{ km} \doteq \frac{6}{10} \text{ mi.}$$

$$1 \text{ mi.} \doteq 1.6 \text{ km}$$

5. 1 mile = 5280 ft.

Remember: If an object maintains its shape, then

$$V = (\text{Area of base})(\text{height})$$

If an object goes to a point (has an APEX), then

$$V = \frac{1}{3}(\text{Area of base})(\text{height})$$

Part I - Multiple Choice (30 marks)

Determine the correct answer to each of the following and place the answer in the space provided on the answer sheet at the end of Part 1.

1. What is 100 in. converted to yards, feet and inches?

(A) 2 yds. 2 ft. 2 in.

(B) 2 yds. 2 ft. 4 in.

(C) 1 yd. 1 ft. 4 in.

(D) 4 yds. 0 ft. 4 in.

$$36 \times 2 = 72 \text{ in} = \underline{2 \text{ yds.}}$$

$$1 \text{ yd} = 36 \text{ in}$$

$$100 - 72 = 28 \text{ in}$$

$$28 \text{ in} = \underline{2 \text{ ft.}} \quad \underline{4 \text{ inches.}}$$

2. Which referent could you use for 1 m?

(A) The width of a computer keyboard

(B) The length of a dinner fork

(C) The length of your stride

(D) The width of a classroom in your school

3. A model of the CN Tower has a scale of 1:300. The height of the model is 25 inches. What is the height of the CN Tower to the nearest foot?

(A) 7500 ft.

(B) 209 ft.

(C) 144 ft.

(D) 625 ft.

$$25 \times 300 = 7500 \text{ inches}$$

$$\frac{7500}{12} = 625 \text{ ft.}$$

4. Convert 1246 yds. to the nearest tenth of a Kilometer.

(A) 1.3 km

(B) 1.1 km

(C) 1.8 km

(D) 2.3 km

$$1 \text{ yd} = 0.9 \text{ m.}$$

$$\therefore 1246 \text{ yds} = 1246 \times 0.9 \text{ m}$$

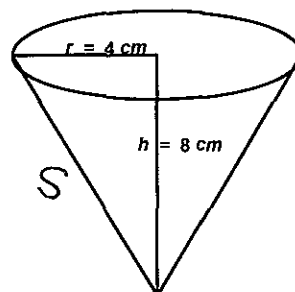
$$= 1121.4 \text{ m}$$

$$= \underline{\underline{1.12 \text{ km.}}}$$

5. What is the lateral area of the cone to the nearest tenth?

- (A) 112.4 cm²
- (B) 100.5 cm²
- (C) 162.7 cm²
- (D) 150.8 cm²

$$\begin{aligned} \text{Lateral Area} &= \pi r s \\ &= \pi (4)(8.9) \\ &= 112.4 \text{ cm}^2 \end{aligned}$$

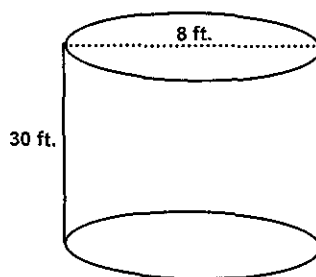


$$\begin{aligned} 4^2 + 8^2 &= S^2 \\ 16 + 64 &= S^2 \\ 80 &= S^2 \\ 8.9 &= S \end{aligned}$$

6. A water tank is the shape of a right cylinder 30 ft. high and 8 ft. in diameter. How much sheet metal was used in its construction?

- (A) 1608.5 ft²
- (B) 1910.1 ft²
- (C) 804.2 ft²
- (D) 854.5 ft²

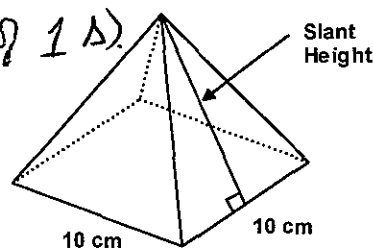
$$\begin{aligned} \text{S.A.} &= 2\pi r h + 2\pi r^2 \\ &= 2\pi (4)(30) \\ &\quad + 2\pi (4)^2 \\ &= 240\pi + 32\pi \\ &= 272\pi \\ &= 854.5 \text{ ft}^2 \end{aligned}$$



7. A square based pyramid with side length 10 cm has a lateral area of 80 cm². What is the slant height of the pyramid?

- (A) 4 cm
- (B) 5 cm
- (C) 8 cm
- (D) 12 cm

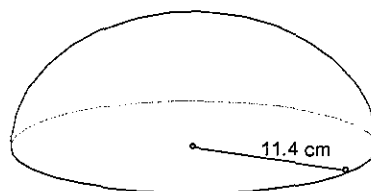
$$\begin{aligned} \frac{80}{4} &= 20 \text{ cm}^2 \text{ (Area of 1 } \Delta) \\ A &= \frac{1}{2} (10)(s) \\ 20 &= 5s \\ \underline{4} &= \underline{s} \end{aligned}$$



8. A hemisphere has radius 11.4 cm. What is the volume of the hemisphere to the nearest tenth of a cubic centimetre?

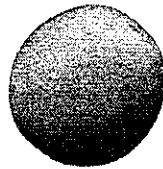
- (A) 6205.9 cm³
- (B) 3102.9 cm³
- (C) 1633.1 cm³
- (D) 1224.8 cm³

$$\begin{aligned} V &= \frac{1}{2} \left(\frac{4}{3} \pi r^3 \right) \\ &= \frac{4}{6} \pi r^3 \\ &= \frac{2}{3} \pi r^3 \\ V &= \frac{2}{3} \pi (11.4)^3 = \underline{\underline{3102.9 \text{ cm}^3}} \end{aligned}$$



9. An orange is peeled and the surface area is found to be 339.8 cm^2 . What is its radius?

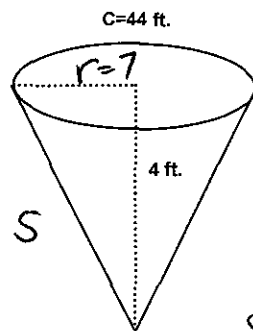
- (A) 16.3 cm
- (B) 13.5 cm
- (C) 7.4 cm
- (D) 5.2 cm



$$\begin{aligned}
 SA &= 4\pi r^2 \\
 339.8 &= 4\pi r^2 \\
 \frac{339.8}{4\pi} &= r^2 \\
 27.04 &= r^2 \\
 \underline{5.2 \text{ cm.}} &= r
 \end{aligned}$$

10. A cone has a circumference of 44 ft and a height of 4 ft. What is its slant height? ($C = 2\pi r$)

- (A) 7.7 ft.
- (B) 8.1 ft.
- (C) 65 ft.
- (D) 60 ft.



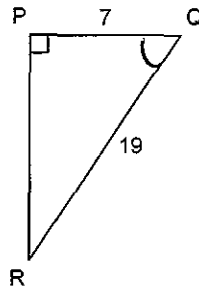
$$\begin{aligned}
 \therefore S &= \sqrt{65} \\
 &= 8.1
 \end{aligned}$$

$$\begin{aligned}
 C &= 2\pi r \\
 44 &= 2\pi r \\
 \frac{44}{2\pi} &= r \\
 7 &= r
 \end{aligned}$$

$$\begin{aligned}
 S^2 &= 7^2 + 4^2 = 49 + 16 \\
 &= 65
 \end{aligned}$$

11. Determine the measure of $\angle Q$ to the nearest tenth of a degree.

- (A) 68.4°
- (B) 69.8°
- (C) 21.6°
- (D) 20.2°



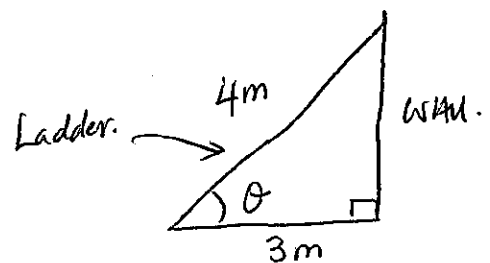
$$\cos Q = \frac{\text{adj}}{\text{hyp}} = \frac{7}{19}$$

$$\begin{aligned}
 \therefore \angle Q &= \cos^{-1}\left(\frac{7}{19}\right) \\
 &= 68.4^\circ
 \end{aligned}$$

12. A 4 m long ladder leans against a building. If the base of the ladder is 3m from the wall, what is the angle of inclination of the ladder?

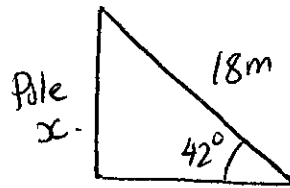
- (A) 48.6°
- (B) 36.9°
- (C) 41.4°
- (D) 53.1°

$$\begin{aligned}
 \cos \theta &= \frac{3}{4} \\
 \theta &= \cos^{-1}(0.75) \\
 \theta &= 41.4^\circ
 \end{aligned}$$



13. A light pole has a 18m guy wire attached to its top. If the angle formed between the ground and the guy wire is 42° , what is the length of the pole?

- (A) 12.0 m
(B) 13.4 m
(C) 26.9 m
(D) 20.0 m



$$\sin 42^\circ = \frac{x}{18}$$

$$x = 12.0$$

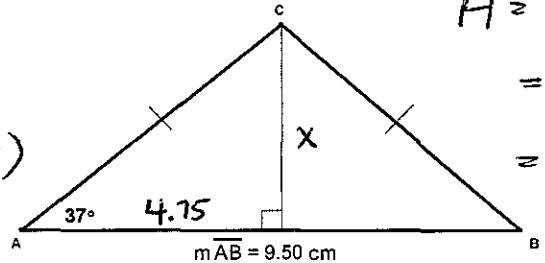
14. What is the area of $\triangle ABC$?

- (A) 6.8 cm^2
(B) 13.5 cm^2
(C) 8.5 cm^2
(D) 17.0 cm^2

$$\tan 37^\circ = \frac{x}{4.75}$$

$$x = (\tan 37^\circ)(4.75)$$

$$x = 3.6 \text{ cm.}$$



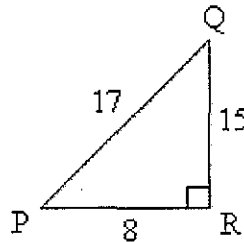
$$A = \frac{1}{2} b \cdot h$$

$$= \frac{1}{2} (9.5)(3.6)$$

$$= 17.1 \text{ cm}^2$$

15. Which of the following statements is true of the diagram below?

- (A) $\cos P = \frac{8}{15}$
(B) $\tan Q = \frac{15}{8}$
(C) $\sin P = \frac{8}{17}$
(D) $\cos Q = \frac{15}{17}$



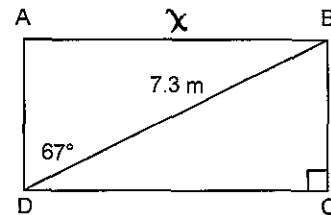
16. Calculate the length (the longer side) of this rectangle to the nearest tenth of a metre.

- (A) 2.9 m
(B) 7.9 m
(C) 6.7 m
(D) 3.1 m

$$\sin 67^\circ = \frac{x}{7.3}$$

$$x = (7.3)(\sin 67^\circ)$$

$$x = 6.7 \text{ cm.}$$



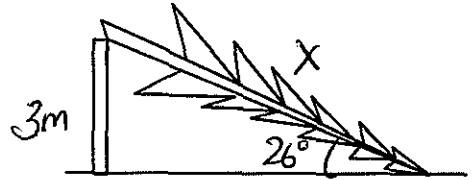
17. A tree breaks off 3m from its base such that the top of the tree just touch the ground. If the top of the tree forms an angle of 26° with the ground, how long was the tree before it broke?

- (A) 6.8 m
- (B) 9.8 m
- (C) 3.3 m
- (D) 6.3 m

$$\sin 26^\circ = \frac{3}{x}$$

$$x = \frac{3}{\sin 26^\circ}$$

$$x = 6.8 \text{ m}$$



$$\therefore \text{Tree} = 3\text{m} + 6.8\text{m} = 9.8\text{m}$$

18. Determine the measure of $\angle D$ to the nearest tenth of a degree.

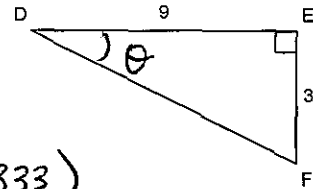
- (A) 18.4°
- (B) 19.5°
- (C) 70.5°
- (D) 71.6°

$$\tan \angle D = \frac{3}{9}$$

$$\tan \angle D = 0.333$$

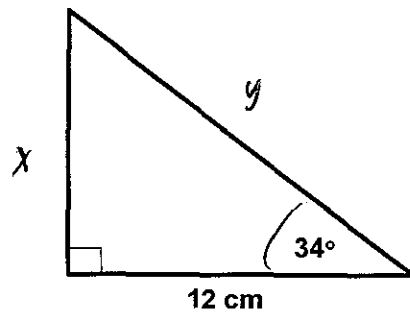
$$\angle D = \tan^{-1}(0.333)$$

$$\angle D = 18.4^\circ$$



19. What is the perimeter of the triangle?

- (A) 22.6 cm
- (B) 34.6 cm
- (C) 30.0 cm
- (D) 44.3 cm



$$\left. \begin{aligned} \text{Perimeter} &= 12 + 8.1 + 14.5 \\ &= 34.6 \text{ cm.} \end{aligned} \right\} \begin{aligned} \tan 34^\circ &= \frac{x}{12} \\ x &= 12(\tan 34^\circ) \\ x &= 8.1 \text{ cm.} \end{aligned}$$

$$\cos 34^\circ = \frac{12}{y}$$

$$y = \frac{12}{\cos 34^\circ}$$

$$y = 14.5 \text{ cm}$$

20. Which of the following numbers is irrational?

(A) 12.235 Rational - Terminating decimal.

(B) $\sqrt[4]{\frac{16}{81}} = \frac{2}{3}$ (Rational)

(C) $\sqrt[3]{\frac{27}{11}} \rightarrow$ Irrational

(D) $\frac{6.9}{3} = 2.3$ Rational

21. Which statement is true?

(A) Some rational numbers are also irrational. F

(B) All rational numbers are integers. F

(C) All natural numbers are whole numbers. T

(D) Some irrational numbers are whole numbers. F

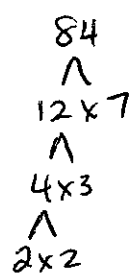
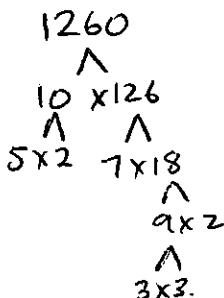
22. Which represents the GCF of 1260 and 84?

(A) $2^2 \cdot 3 \cdot 7$

(B) $2 \cdot 3 \cdot 7$

(C) $2^2 \cdot 3^2 \cdot 7$

(D) $2^2 \cdot 3^2 \cdot 5 \cdot 7$



$$\begin{array}{l}
 1260 = 2^2 \cdot 3^2 \cdot 5 \cdot 7 \\
 84 = 2^2 \cdot 3 \cdot 7 \\
 \text{GCF} = 2^2 \cdot 3 \cdot 7
 \end{array}$$

23. Which set of prime factors would represent a perfect cube?

(A) $2 \cdot 3 \cdot 7 \cdot 2 \cdot 3 \cdot 7 \rightarrow (2 \cdot 3 \cdot 7)(2 \cdot 3 \cdot 7)$ Perfect Square.

(B) $3 \cdot 3 \cdot 3 \cdot 3 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \rightarrow (3 \cdot 3 \cdot 7 \cdot 7)(3 \cdot 3 \cdot 7 \cdot 7)$ Perfect Square.

(C) $4 \cdot 4 \cdot 4 \cdot 5 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 7 \rightarrow (4 \cdot 5 \cdot 7)(4 \cdot 5 \cdot 7)(4 \cdot 5 \cdot 7)$ Perfect Cube

(D) $3 \cdot 4 \cdot 5 \cdot 3 \cdot 4 \cdot 5$

$\rightarrow (3 \cdot 4 \cdot 5)(3 \cdot 4 \cdot 5)$ Perfect Square.

24. What is $2\sqrt[4]{3}$ as an entire radical?

(A) $\sqrt[4]{6}$

(B) $\sqrt[4]{24}$

(C) $\sqrt[4]{32}$

(D) $\sqrt[4]{48}$

$$\begin{aligned} 2\sqrt[4]{3} &= \sqrt[4]{16} \times \sqrt[4]{3} \\ &= \sqrt[4]{48} \end{aligned}$$

25. Identify the index of $\sqrt[3]{2^7}$.

(A) 2^7

(B) 3

(C) 7

(D) 2

26. A cube has a volume of 128. What is the length of the side as a mixed radical in simplest form?

(A) $4\sqrt[3]{2}$

(B) $3\sqrt[3]{2}$

(C) $2\sqrt[3]{64}$

(D) $3\sqrt{2}$

$$\begin{aligned} \sqrt[3]{128} &= \sqrt[3]{64} \times \sqrt[3]{2} \\ &= 4\sqrt[3]{2} \end{aligned}$$

27. Evaluate: $\sqrt[3]{-\left(\frac{8}{27}\right)} = \frac{\sqrt[3]{-8}}{\sqrt[3]{27}} = \frac{-2}{3}$

(A) $\frac{2}{3}$

(B) $\frac{3}{2}$

(C) $-\frac{2}{3}$

(D) $-\frac{3}{2}$

28. What is $(3)^{\frac{4}{5}}$ written as a radical?

(A) $\sqrt[5]{3^4}$

(B) $\sqrt[4]{3^5}$

(C) $\sqrt[5]{4^3}$

(D) $\sqrt[3]{4^5}$

29. What is $\sqrt[3]{(-5)^7}$ written as a power?

(A) $(-5)^{\frac{7}{3}}$

(B) $-5^{\frac{7}{3}}$

(C) $(-5)^{\frac{3}{7}}$

(D) $-5^{\frac{3}{7}}$

30. Which is equivalent to $\left(\frac{2}{3}\right)^{-4}$?

(A) $\left(-\frac{16}{81}\right)$

(B) $\left(-\frac{81}{16}\right)$

(C) $\left(\frac{16}{81}\right)$

(D) $\left(\frac{81}{16}\right)$

$$\left(\frac{2}{3}\right)^{-4} = \frac{2^{-4}}{3^{-4}} = \frac{3^4}{2^4} = \frac{81}{16}$$

Part I Answer Sheet

Name: _____

Solutions

1. B

2. C

3. D

4. B

5. A

6. D

7. A

8. B

9. D

10. B

11. A

12. C

13. A

14. D

15. D

16. C

17. B

18. A

19. B

20. C

21. C

22. A

23. C

24. D

25. B

26. A

27. C

28. A

29. A

30. D

Name: _____

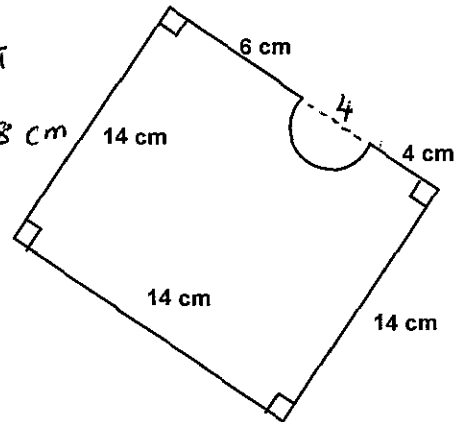
Part II - Short Answer Questions (9 Marks)

Answer each of the following questions in the space provided. Show all workings where appropriate as marks will be given for partially correct answers.
(3 marks each)

1. Find the perimeter of the following figure. Express your answer to the nearest tenth of an inch. ($C = 2\pi r$)

Semicircle · $C = \frac{2\pi r}{2} = \frac{2\pi(2)}{2} = 2\pi$
 $= 6.28 \text{ cm}$

Perimeter = $(14)(3) + 6 + 4 + 6.28$
 $= 42 + 10 + 6.28$
 $= 58.28 \text{ cm.}$



To the nearest tenth of an inch

$1 \text{ cm} = \frac{4}{10} = 0.4 \text{ inches.}$ $\therefore 58.28 \text{ cm} \times 0.4$
 $= \underline{\underline{23.3 \text{ inches}}}$

2. Use Prime factorization to find the LCM of 84 and 360.

$$\begin{array}{c} 84 \\ \wedge \\ 12 \times 7 \\ \wedge \\ 4 \times 3 \\ \wedge \\ 2 \times 2 \end{array}$$

$84 = 2^2 \cdot 3 \cdot 7$

$$\begin{array}{c} 360 \\ \wedge \\ 10 \times 36 \\ \wedge \quad \wedge \\ 5 \times 2 \quad 9 \times 4 \\ \quad \wedge \quad \wedge \\ \quad 3 \times 3 \quad 2 \times 2 \end{array}$$

$360 = 2^3 \cdot 3^2 \cdot 5$

$\text{LCM} = 2^3 \cdot 3^2 \cdot 5 \cdot 7 = 8 \times 9 \times 5 \times 7$
 $= 2520.$

Name: _____

3. Is $-9^{\frac{1}{2}}$ equal to $(-9)^{\frac{1}{2}}$? Explain your reasoning.

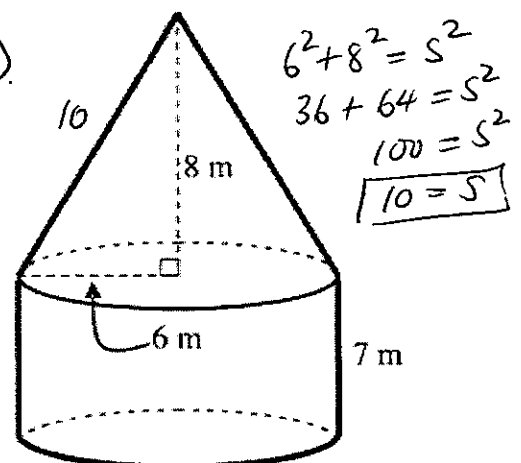
No. $-9^{\frac{1}{2}} = -1 \cdot 9^{\frac{1}{2}} = -1 \cdot \sqrt{9} = -1(3) = -3$
 $(-9)^{\frac{1}{2}} = \sqrt{-9}$ which is undefined!

Part III. Long Answer. (21 Marks)

Answer each of the following questions in the space provided. Show all workings where appropriate as marks will be given for partially correct answers.

1. a) If a can of paint covers 30 m^2 , determine how many cans of paint is needed to paint the outside (including the bottom) of this composite object. (3 marks)

S.A. Cone (only need lateral Area).
 $S.A = \pi r s$
 $= \pi (6)(10)$
 $= 60\pi$
 $= 188.5 \text{ m}^2$



S.A. Cylinder. (Need Curved part + 1 Circle).

$S.A = 2\pi r h + \pi r^2$
 $= 2\pi (6)(7) + \pi (6)^2$
 $= 84\pi + 36\pi$
 $= 120\pi$
 $= 377 \text{ m}^2$

Total S.A.
 $188.5 + 377$
 $= 565.5 \text{ m}^2$

Point: $\frac{565.5}{30} = 18.85$
 \therefore Need 19 cans

Name: _____

b) Calculate the volume of the following figure. (3 marks)

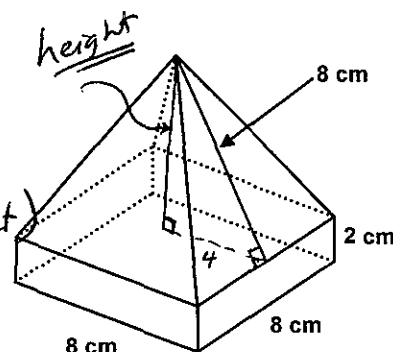
Volume of rectangular prism.

$$L \times W \times h = 8 \times 8 \times 2 = \underline{128 \text{ cm}^3}$$

Volume of pyramid. = $\frac{1}{3}$ (Area of base) (height)

$$= \frac{1}{3} (8 \times 8) (6.9)$$

$$= \underline{147.2 \text{ cm}^3}$$

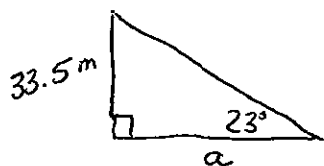


$$\begin{aligned} h^2 + 4^2 &= 8^2 \\ h^2 &= 64 - 16 \\ h^2 &= 48 \\ h &= 6.9 \text{ cm.} \end{aligned}$$

$$\begin{aligned} \text{Total Volume} &= 128 + 147.2 \\ &= \underline{275.2 \text{ cm}^3} \end{aligned}$$

2. A lighthouse keeper is spotted from a fishing boat at an angle of elevation of 23° . At the same time a person on a sailboat spots the lighthouse keeper at an angle of elevation of 9° . If the lighthouse keeper is 33.5 m above the water, how far apart are the two vessels? (4 marks)

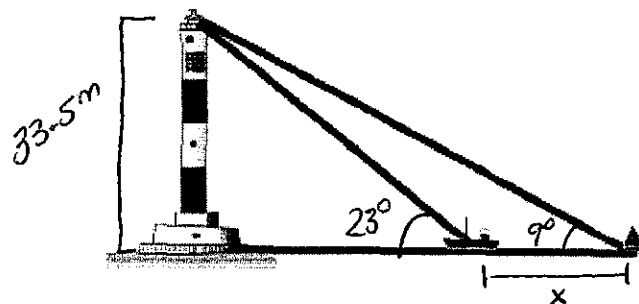
- ① First determine how far the fishing boat is from the lighthouse.



$$\tan 23^\circ = \frac{33.5}{a}$$

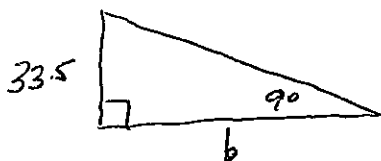
$$\therefore a = \frac{33.5}{\tan 23^\circ}$$

$$\therefore a = 78.9 \text{ m.}$$



$$\begin{aligned} \therefore x &= b - a \\ &= 211.5 - 78.9 \\ &= \underline{132.6 \text{ m}} \end{aligned}$$

- ② Determine how far the sailboat is from the lighthouse



$$\tan 9^\circ = \frac{33.5}{b}$$

$$\therefore b = \frac{33.5}{\tan 9^\circ} = 211.5 \text{ m.}$$

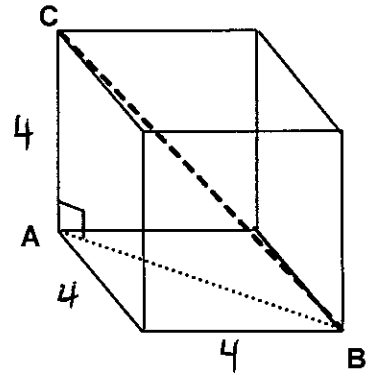
Boats are
132.6 m
Apart.

Name: _____

3. A cube has a volume of 64 cm^3 . Determine the diagonal distance through the cube from one corner to the opposite corner (\overline{BC}). Express your answer as a mixed radical in simplest form. Hint: find the length of \overline{AB} first. (5 marks)

$$\text{Side length of cube} = \sqrt[3]{64} = 4$$

$$\begin{aligned} 4^2 + 4^2 &= (\overline{AB})^2 \\ 16 + 16 &= (\overline{AB})^2 \\ 32 &= (\overline{AB})^2 \\ \sqrt{32} &= \overline{AB} \end{aligned}$$



$$\begin{aligned} (\overline{AB})^2 + (\overline{AC})^2 &= (\overline{BC})^2 \\ (\sqrt{32})^2 + (4)^2 &= (\overline{BC})^2 \\ 32 + 16 &= (\overline{BC})^2 \\ 48 &= \overline{BC}^2 \\ \sqrt{48} &= \overline{BC} \end{aligned}$$

$$\begin{aligned} \overline{BC} &= \sqrt{16} \times \sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

4. Evaluate. Show all workings. (5 marks)

$$(a) \frac{1}{(-32)^{\frac{2}{5}}}$$

$$\begin{aligned} &= (-32)^{\frac{2}{5}} \\ &= (\sqrt[5]{-32})^2 \\ &= (-2)^2 \\ &= 4 \end{aligned}$$

$$(b) \left(\frac{27}{64}\right)^{-\frac{2}{3}} = \frac{27^{-2/3}}{64^{-2/3}}$$

$$\begin{aligned} &= \frac{64^{2/3}}{27^{2/3}} \\ &= \frac{(\sqrt[3]{64})^2}{(\sqrt[3]{27})^2} = \frac{4^2}{3^2} = \frac{16}{9} \end{aligned}$$