Sample Exam (May 9th)

1. $$\lim_{x \to -5} \frac{(x+5)(x-5)}{x^2(x+5)+5(x+5)}$$

$$\lim_{x \to -5} \frac{\cancel{(x+5)}(x-5)}{\cancel{(x+5)}(x^2+5)}$$

$$= \frac{-10}{30} = \boxed{\frac{-1}{3}}$$

2. $$\lim_{x \to 2} \frac{(-3+\sqrt{4x+1})(-3-\sqrt{4x+1})}{(10-5x)\cdot(-3-\sqrt{4x+1})}$$

$$= \lim_{x \to 2} \frac{9-(4x+1)}{(10-5x)(-3-\sqrt{4x+1})}$$

$$= \lim_{x \to 2} \frac{4\cancel{(2-x)}}{5\cancel{(2-x)}(-3-\sqrt{4x+1})}$$

$$= \frac{\cancel{4}\,2}{5(\cancel{-6})_3} = \boxed{\frac{2}{-15}}$$

3. $$\lim_{x \to 3^+} \frac{-\cancel{(3-x)}^{(-1)}}{\cancel{(x-3)}(x-3)}$$

$$= \boxed{\infty}$$

4. $$\lim_{x \to -\infty} \frac{\frac{1}{x}(2x-1)}{\frac{1}{x}(x-\sqrt{9x^2+7})}$$

$$\lim_{x \to -\infty} \frac{2-\frac{1}{x}}{1+\sqrt{\frac{9x^2}{x^2}+\frac{7}{x^2}}}$$

$$\lim_{x \to -\infty} \frac{2-\frac{1}{x}}{1+\sqrt{9+\frac{7}{x^2}}}$$

$$= \frac{2}{4} = \boxed{\frac{1}{2}}$$

2. a) $y = \csc(x^2e^x)$

$y' = -\csc(x^2e^x)\cot(x^2e^x)\cdot[x^2e^x + 2xe^x]$

$y' = -x^2e^x\csc(x^2e^x)\cot(x^2e^x) - 2xe^x\csc(x^2e^x)\cot(x^2e^x)$

$y' = -xe^x\csc(x^2e^x)\cot(x^2e^x)[x+2]$

b) $y = \dfrac{\tan^{-1}(x)}{1+x^2}$

$y' = \dfrac{(1+x^2)\frac{1}{1+x^2} - 2x(\tan^{-1}x)}{(1+x^2)^2}$

$y' = \dfrac{1 - 2x\tan^{-1}x}{(1+x^2)^2}$

c) $y = \sin(e^{x^4})$

$y' = \cos(e^{x^4})\cdot e^{x^4}\cdot 4x^3$

d) $y = x^{\sin x}$ re-write eqn 1st!

$\ln y = \sin x \cdot \ln x$

$\frac{1}{y}\cdot y' = \sin x \cdot \frac{1}{x} + \cos x \cdot \ln x$

$y' = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \ln x\right]$

last one has the bracket missing... it would then be the correct ans.

e) $xy^2 = 2x - y$

$x\cdot 2yy' + 1y^2 = 2 - 1y'$

$2xyy' + y' = 2 - y^2$

$y'(2xy+1) = 2 - y^2$

$y' = \dfrac{2-y^2}{2xy+1}$

3. $f(x) = \begin{cases} \dfrac{4(x+2)}{(x-4)(x+2)} & , x \leq 0 \\ x^2 - 2x & , x > 0 \end{cases}$

Defn $\rightarrow$ 3 parts

① $f(0) = -1$ so $f(0)$ exist

② $\lim\limits_{x \to 0^-} f(x) = \boxed{-1}$

$\lim\limits_{x \to 0^+} f(x) = \boxed{0}$

so $\lim\limits_{x \to 0} f(x)$ = dne

$\therefore$ f(x) is not continuous at 0

This would be a jump discontinuity which is non-removable.

b) Def'n

① $f(-2) =$ dne

f(x) has a removable discontin. (hole) at $x = -2$.

- f(x) does not have any other discontinuities.

4. $f(x) = \dfrac{2x}{5-x}$, $f(x+h) = \dfrac{2x+2h}{5-x-h}$

So $y' = \lim\limits_{h \to 0} \left[\dfrac{\dfrac{2x+2h}{5-x-h} - \dfrac{2x}{5-x}}{\frac{h}{1}} \right]$

$= \lim\limits_{h \to 0} \dfrac{(2x+2h)(5-x) - 2x(5-x-h)}{h(5-x)(5-x-h)}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{10x} - \cancel{2x^2} + 10h - \cancel{2xh} - \cancel{10x} + \cancel{2x^2} + \cancel{2xh}}{h(5-x)(5-x-h)}$

$= \lim\limits_{h \to 0} \dfrac{10\cancel{h}}{\cancel{h}(5-x)(5-x-h)}$

$= \lim\limits_{h \to 0} \dfrac{10}{(5-x)(5-x-h)}$

$= \boxed{\dfrac{10}{(5-x)^2}}$

b) find eqn of tangent line

$$y = \frac{2x}{5-x} \quad \text{at } x = 4$$

step 1 → ordered pr!

$(4, 8)$

step 2 → get deriv & plug in $x=4$

$$y' = \frac{(5-x)(2) - (-1)(2x)}{(5-x)^2}$$

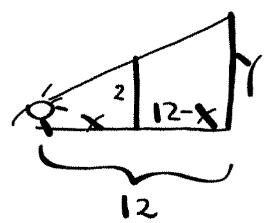
$$y'\Big|_4 = \frac{(1)(2) + 8}{1}$$

$$= \boxed{10}$$

step 3 → ans. the ques.

$$y - 8 = 10(x - 4)$$

5.



$\frac{dx}{dt} = 1$ m/s

find: $\frac{dy}{dx}$ when $x=8$

$$\frac{2}{y} = \frac{x}{12}$$

find

$$xy = 24$$

$$x\frac{dy}{dt} + 1 \cdot \frac{dx}{dt} \cdot y = 0$$

detour

$$\frac{2}{y} = \frac{8}{12}$$

$$8y = 24$$

$$y = 3$$

$$8 \cdot \frac{dy}{dt} + 1 \cdot 1 \cdot 3 = 0$$

$$8\frac{dy}{dt} = -3$$

$$\frac{dy}{dt} = -\frac{3}{8} \text{ m/s}$$