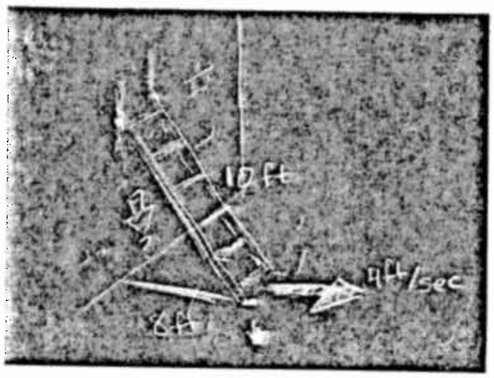
Name: ____________________

Falling Ladder

A 10 ft. ladder leans against a wall. It is slipping outward from the base at a rate of 4 ft/sec. What is the rate of change in the height that the ladder reaches up on the wall at the instant that the base of the ladder is 8 ft. from the wall?



$x^2+y^2=10^2$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

$2(8)(4)+2(6)\frac{dy}{dt}=0$

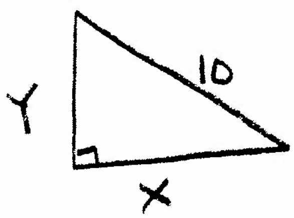
* When $x=8$

$x^2+y^2=100$

$64+y^2=100$

$y=\sqrt{36}$

$y=6$



$\frac{dx}{dt}=4$

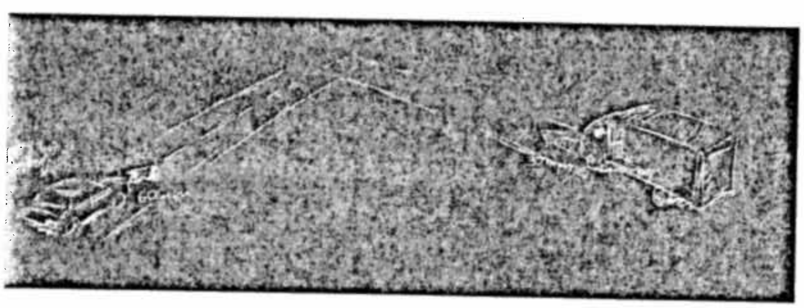
$\left.\frac{dy}{dt}\right|_{x=8}$

$\frac{dy}{dt}=\frac{-64}{12}$

$\frac{dy}{dt}=-\frac{16}{3}$ ft/sec

Approaching Cars

Two cars are approaching an intersection? Car Lundy is driving at 60 mph while Car Connors is dring at 30 mph. At the time when Lundy is 8 miles from the intersection, Connors is 6 miles from the intersection. How is the distance between them changing at that time?



$L^2+c^2=x^2$

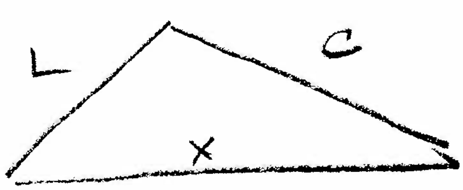
$\cancel{2}L\frac{dL}{dt}+\cancel{2}c\frac{dc}{dt}=\cancel{2}x\frac{dx}{dt}$

$(8)(-60)+(6)(-30)=(10)\frac{dx}{dt}$

$-480-180=10\frac{dx}{dt}$

$\frac{-660}{10}=\frac{dx}{dt}$

$\boxed{-66 \text{ mph}}$



Need

$\left.\frac{dx}{dt}\right|_{L=8,\ c=6}$

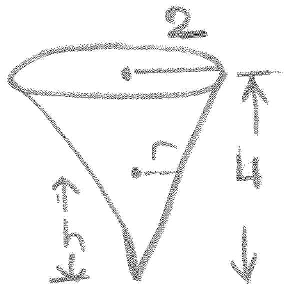
$\frac{dL}{dt}=-60$, $\frac{dc}{dt}=-30$

when $L=8$ & $c=6 \Rightarrow x=10$

Cones

Water is poured into a cone shaped cup at a rate of 1 ~~cm|~~ cm^3 / sec. How is the depth of the water changing at the instant when its depth measures 2 cm.?

depth = 4, diameter = 4



$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2(h)$$

$$V = \frac{\pi}{12}h^3$$

$$\frac{dv}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt}$$

$$\frac{dv}{dt} = 1 \qquad \left.\frac{dh}{dt}\right|_{h=2}$$

$$1 = \frac{\pi}{4}(2)^2\left(\frac{dh}{dt}\right)$$

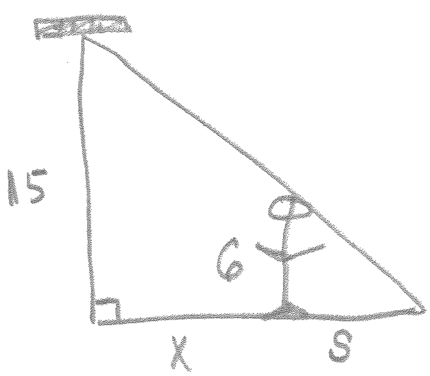
$$\frac{r}{h} = \frac{1}{2} \ni \boxed{r = \frac{h}{2}}$$

$$\frac{1}{\pi}\ \text{cm/sec} = \frac{dh}{dt}$$

Shadows

Rebecca is walking away from a lamp post at a rate of 3 ft/sec. If Rebecca is 6 ft. tall and the lamp post is 15 ft. high, what rate is her shadow changing when she is 10 ft. from the post?

× similar Δ's



$$\frac{6}{15} = \frac{S}{X+S}$$

$$\frac{2}{5} = \frac{S}{X+S}$$

$$2X + 2S = 5S$$

$$2X = 3S$$

$$2\frac{dx}{dt} = 3\frac{ds}{dt}$$

$$\frac{dx}{dt} = 3$$

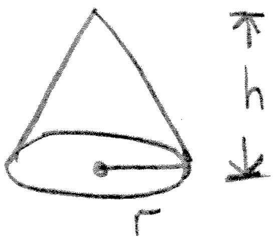
Need:

$$\left.\frac{ds}{dt}\right|_{X=10}$$

$$2(3) = 3\frac{ds}{dt}$$

$$2\ \text{ft/sec} = \frac{ds}{dt}$$

5. Sand falling from a chute forms a conical pile whose height is always equal to $4/3$ of the radius of the base. How fast is the volume changing when the radius of the base is 3 ft and is increasing at a rate of 3 in/min?



$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi r^2 \left(\frac{4}{3} r\right)$$

$$V = \frac{4}{9}\pi r^3$$

$h = \frac{4}{3} r$, $\frac{dr}{dt} = 3$

$$\frac{dV}{dt} = \frac{12}{9}\pi r^2 \left(\frac{dr}{dt}\right)$$

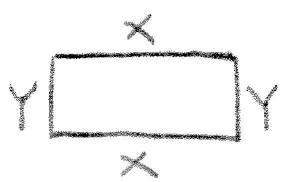
NEED $\left.\frac{dV}{dt}\right|_{r=36}$

$$= \frac{4}{3}\pi (36)^2 (3)$$

$$= \cancel{5184}\pi \text{ in}^3/\text{MIN.} \quad 16286$$

$5184\pi \text{ IN}^3/\text{MIN}$

6. Two parallel sides of a rectangle are being lengthened at a rate of 2 in/sec, while the other 2 sides are being shortened in such a way that the figure remains a rectangle with a constant area of 50 in^2. What is the rate of change in the perimeter when the lengthening side is 5 in.? Is the perimeter increasing or decreasing?



$$P = 2x + 2y$$

$$P = 2x + 2\left(\frac{50}{x}\right)$$

$$P = 2x + \frac{100}{x}$$

$xy = 50$, $\frac{dx}{dt} = 2$

$y = \frac{50}{x}$

$$\frac{dP}{dt} = 2 + -\frac{100}{x^2}\left(\frac{dx}{dt}\right)$$

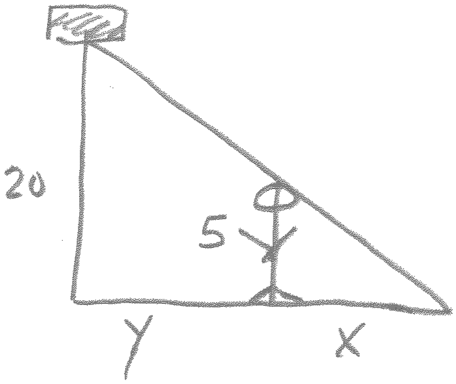
Need $\left.\frac{dP}{dt}\right|_{x=5}$

$$= 2 - \frac{100}{(5)^2}(2)$$

$$= 4 - 8 = -4 \text{ IN/sec}$$

$\star$ DECREASING

7. A boy 5 ft tall walks at a rate of 4 ft/sec directly away from a street light which is 20 feet above the street. At what rate is his shadow changing? Is the length of the shadow increasing or decreasing?



USE SIMILAR TRIANGLES

$$\frac{5}{20} = \frac{x}{x+y} \Rightarrow \frac{1}{4} = \frac{x}{x+y}$$

$$x + y = 4x$$

$$y = 3x$$

$$\frac{dy}{dt} = 3\frac{dx}{dt}$$

$$\frac{4}{3} = \frac{3}{3}\frac{dx}{dt}$$

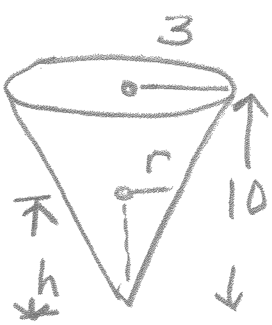
$$\frac{dx}{dt} = \frac{4}{3}$$

INCREASING

x = SHADOW

$\frac{dy}{dt} = 4$ Need $\frac{dx}{dt}$

8. Water is being withdrawn from a conical reservoir 3 ft in radius and 10 ft deep at a rate of 4 ft^3/min. How fast is the water level falling when the depth of water in the reservoir is 6 feet?



Need: $\left.\frac{dh}{dt}\right|_{h=6}$

$\frac{dv}{dt} = -4$

$\frac{r}{h} = \frac{3}{10}$

$\therefore r = \frac{3h}{10}$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{3h}{10}\right)^2(h)$$

$$V = \frac{3\pi h^3}{100}$$

$$\frac{dv}{dt} = \frac{9\pi h^2}{100}\frac{dh}{dt}$$

$$-4 = \frac{9\pi(6)^2}{100}\frac{dh}{dt}$$

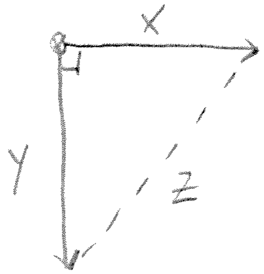
$$-4 = \frac{324\pi}{100}\frac{dh}{dt}$$

$$\frac{-400}{324\pi} = \frac{dh}{dt}$$

$$\frac{-100}{81\pi} = \frac{dh}{dt}$$

ft/min

9. A train, starting at 11 am, travels east at 45 mph while another, starting at noon from the same point, travels south at 60 mph. How fast are they moving away from each other at 3 pm



$x^2 + y^2 = z^2$

$\cancel{2}x \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt} = \cancel{2}z \frac{dz}{dt}$

$180(45) + 180(60) = 254.6 \frac{dz}{dt}$

$\frac{8100 + 10800}{254.6} = \frac{dz}{dt}$

$\overset{74.}{43.4}$ km/hr $= \frac{dz}{dt}$

at 3 o'clock

$x = 4 \times 45 = 180$

$y = 3 \times 60 = 180$

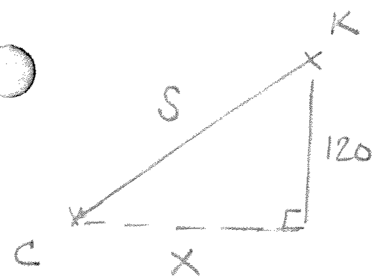
$z = \sqrt{180^2 + 180^2}$

$z \approx 254.6$

$\frac{dx}{dt} = 45, \ \frac{dy}{dt} = 60$

$\left.\frac{dz}{dt}\right|$ at 3'oclock

10. A child is flying a kite 120 ft above the ground. She lets out the 2.5 ft of string per second. If we assume that there is no sag in the string, at what speed is the kite moving when there is 130 ft of string out?



$s^2 = x^2 + 120^2$

$\cancel{2}s \frac{ds}{dt} = \cancel{2}x \frac{dx}{dt}$

$130(2.5) = 50\left(\frac{dx}{dt}\right)$

$\frac{325}{50} = \frac{dx}{dt} \rightarrow \frac{dx}{dt} = 6.5$ ft/sec

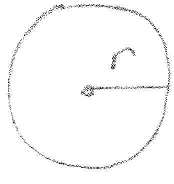
$x^2 = s^2 - 120^2$

$x = \sqrt{130^2 - 120^2}$

$x = 50$

$\frac{ds}{dt} = 2.5, \ \left.\frac{dx}{dt}\right|_{s=130}$

11. Oil is spilled from a ruptured tanker and spreads out in a circle whose area increases at a constant rate of 6 mi^2/hr. How fast is the radius of the spill increasing when the area is 9 mi^2?



$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$6 = 2\pi\left(\sqrt{\frac{9}{\pi}}\right)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{6}{2\pi\left(\sqrt{\frac{9}{\pi}}\right)} \approx$

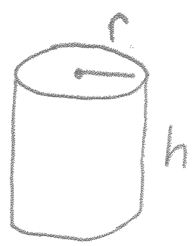
$* \quad 9 = \pi r^2$

$r = \sqrt{\frac{9}{\pi}}$

$\frac{dA}{dt} = 6 \quad \left.\frac{dr}{dt}\right|_{A=9}$

12. You are stood 500 ft from a hot air balloon when it is launched. The balloon ascends vertically . The angle made between the ground and your line of vision is changing at a rate of 0.2 radians per min. How fast is the height changing whan this angle is $\frac{\pi}{4}$?

13. The radius of a right circular cylinder is increasing at a rate of 2 in/min and the height is decreasing at a rate of 3 in/min. At what rate is the volume changing when the radius is 8 in and the height is 12 in? Is the volume increasing or decreasing?



$V = \pi r^2 h$ * Product Rule

$\frac{dV}{dt} = (2\pi r)\left(\frac{dr}{dt}\right)(h) + \pi r^2 \frac{dh}{dt}$

WE KNOW: $\frac{dr}{dt} = 2$, $\frac{dh}{dt} = -3$

$\frac{dV}{dt} = 2\pi(8)(2)(12) + \pi(8)^2(-3)$

We Need: $\left.\frac{dV}{dt}\right|_{r=8,\ h=12}$

$\frac{dV}{dt} = 192\pi$ IN^3/MIN

*14. A beacon makes one revolution every 10 seconds and is anchored on a ship located 4 km from a straight shoreline. How fast is the beam moving along the shoreline when it makes an angle of $\frac{\pi}{4}$ radians with the shore?