Optimisation Problems — Examples

1. The sum of two non-negative numbers is 6. What is the largest possible value of the sum of their squares?

2. A farmer wants to fence in a rectangular plot of land, the area of which is 2400 m^2. She wants to use additional fencing to build an internal divider fence parallel to two of the boundary sections. What is the minimum total length of fencing that this project will require?

3. A farmer with 100 feet of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

4. A closed cylindrical can is to hold 1 litre of liquid. How should we choose the height and radius to minimise the amount of material needed to manufacture the can?

5. An open-topped box is to have a volume of 20 m^3. All the sides of the box are rectangular, and its length is to be twice its width. Material for the base of the box costs $12 per square metre, while material for the sides costs $4 per square metre. Find the cost of the materials for the cheapest such box.

6. Marshall Wyatt Earp is closing in fast on that infamous outlaw, Johnny Ringo. But Ringo knows that there's a hiding place on the opposite side of the San Pedro River, 5 miles downstream. All along this stretch, the river is 1 mile wide. Ringo can swim at 2 miles per hour and run at 3 miles per hour. In order to reach the hideout, he could swim directly across the river and then run all the way to the hideout, swim directly to the hideout, or swim to some point in-between and run the remainder of the distance. To one decimal place, where should Ringo come ashore in order to reach the hiding place as quickly as possible?