1. A rectangular sheet of cardboard measures 24 cm by 18 cm. Congruent squares are cut from the corners of the sheet and the sides are folded to create a rectangular prism. Write a function to model the volume of the prism and use it to algebraically determine the maximum volume of the prism.

2. An oil company wishes to reduce its costs by designing the most economical cylindrical barrels for shipping its oil. The industry standard uses 1.8 m² of material to manufacture each barrel. Using this amount of material, what dimensions would maximize the volume of the barrel?

3. A farmer has 1000 metres of wooden fencing, and she wishes to fence off a rectangular plot of land bordered by a river, so that she does not require fencing on the side of the rectangle adjacent to the water. Find the dimensions of the rectangular plot with the greatest possible enclosed area.

4. The sum of two non-negative numbers is 6. What is the largest possible value of the sum of their squares?

5. A farmer with 100 feet of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

6. A Norman window consists of a rectangle with a semicircle mounted on top. Find the dimensions of the window with the largest area if its perimeter is 10 m.