Mathematics 3208 Name: __________________
2.8 Continuity (Part 1)

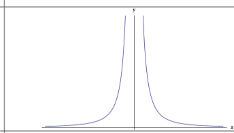
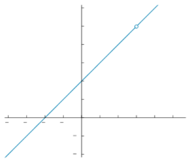
When examining the graph of a function, it is very easy to tell if the function is continuous or not:

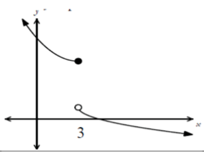
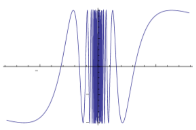
if the function can be drawn without lifting the pencil from the page, the function is said to be ______________________________.

if the pencil must be lifted from the page to draw the function, then the function is _____________________________ .

We will learn a more precise definition later, which will allow us to determine whether a function is continuous based on its equation alone.

There are several types of discontinuities: infinite, removable, jump, and oscillating.

Type of Discontinuity	Example	Notes
Infinite		The one sided limits of the function at $x = 0$ are infinite; graphically this situation corresponds to a vertical asymptote
Removable		$f(x)$ is not defined at the point $x = 3$; a value can be assigned to $f(3)$ to make the extended function continuous at $x = 3$.

Jump		The left and right hand limits at $x = 3$ are not equal. Therefore, the function has no limit at that point.
Oscillating		The function $f(x)$ is not defined at $x = 0$ so it is not continuous at $x = 0$. The function also oscillates between -1 and 1 as x approaches 0. Therefore, the limit does not exist.

Formal Definition of a Limit

Three conditions must be met for a function to be continuous at a particular value a:

If even one of these conditions is not met, then the function is discontinuous.

Example 1: At which numbers is f discontinuous? Why?

Solution:

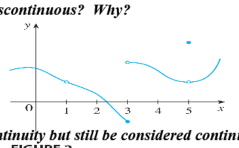


FIGURE 2

Continuity from the Left/Right

A function may have a point of discontinuity but still be considered continuous from either the left or the right.

A function f is continuous from the ______________ at a number a if $\lim_{x\to a^+} f(x) = f(a)$

$\lim_{x\to a^-} f(x) = f(a)$

A function f is continuous from the ______________ at a number a if

In figure 2 above, we see that although f has a point of discontinuity at x=3, we can say that f is continuous from the left at 3, because $\lim_{x\to 3^-} f(x) = f(3)$ ***.However, f is not continuous from the right at 3, because*** $\lim_{x\to 3^+} f(x) \neq f(3)$

Continuity on Intervals

Using the idea of continuity from the left/right, we can identify certain intervals on which a discontinuous function f is continuous.

Example 2: Identify the intervals on which f is continuous.

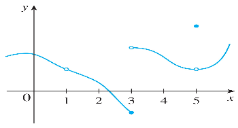
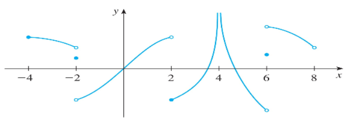


FIGURE 2

1. Using the graph below,
   - identify all x-values where $f(x)$ is discontinuous,
   - use the formal definition of continuity to explain *why* each point is discontinuous,
   - for each identified point of discontinuity, determine whether $f(x)$ is continuous from the right, or from the left, or neither,
   - state the intervals on which $f(x)$ is continuous.

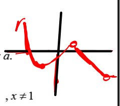


2. Sketch the graph of a function f that is continuous except for the stated discontinuity.

   (A) Discontinuities at -1 and 4, but continuous from the left at -1 and from the right at 4.

   (B) Neither left nor right continuous at -2, continuous only from the left at 2.

3. Determine whether the function is continuous/discontinuous at the given number a.



(A) $f(x)=\begin{cases}\frac{1}{x+2} & ,x\neq -2\\ 1 & ,x=-2\end{cases}$ (B) $a=1$, $f(x)=\begin{cases}\frac{x^2-x}{x^2-1} & ,x\neq 1\\ 1 & ,x=1\end{cases}$

(C) $f(x)=\begin{cases}x+5 & ,x<-2\\ 1 & ,x=-2\\ 1-x & ,x>-2\end{cases}$ (D) $f(x)=\begin{cases}x^2+2x-3 & ,x\leq 1\\ -2x+3 & ,1<x\leq 4\\ -5 & ,x>4\end{cases}$

Check $x=1$

$\lim_{x\to 1^+}(-2x+3)$
$= -2(1)+3$
$= 1$

$\lim_{x\to 1^-}(x^2+2x-3)$
$= (1)^2+2(1)-3$
$= 0$

$f(1)=(1)^2+2(1)-3$
$= 0$

Observation
→ Discontinuous at $x=1$
→ Jump
→ Continuous from Left at $x=1$

at $x = 4$

$\lim_{x\to 4^+}(-5)$ $= -5$

$\lim_{x\to 4^-}(-2x+3) = -5$

$f(4) = -2x+3 = -5$

$\therefore$ Continuous at 4

4. Determine the x-values (if any) at which f is not continuous. Which of the discontinuities are removable?

(A) $f(x)=x^2-2x+1$

Continuous Everywhere $(-\infty,\infty)$

(B) $f(x)=\frac{x}{x^2-x} = \frac{x}{x(x-1)}$

$f(x) = \frac{1}{x-1}$

$\boxed{\text{Check } x=0 \text{ & } x=1}$

$x=0$

$\lim_{x\to 0^+}\left(\frac{1}{x-1}\right) = -1$

$\lim_{x\to 0^-}\left(\frac{1}{x-1}\right) = -1$

$f(0) = \frac{0}{0}$ undefined

* Removable discontinuity at $x=0$

(C) $f(x)=\frac{x+2}{x^2-3x-10} = \frac{(x+2)}{(x+2)(x-5)}$

$f(x) = \frac{1}{x-5}$

$x=-2$ & $x=5$

Removable   VA. $\therefore$ Infinite

(D) $f(x)=\frac{|x+2|}{x+2}$

$|x+2| = \begin{cases} (x+2), & \text{if } x \geq -2 \\ -(x+2), & \text{if } x < -2 \end{cases}$

5. Find the value of a so that the function is continuous.

(A) $f(x)=\begin{cases} x^2-1, & x<3 \\ 2ax, & x\geq 3 \end{cases}$

(B) $f(x)=\begin{cases} 2x+3, & x\leq 2 \\ ax+1, & x>2 \end{cases}$

(C) $f(x)=\begin{cases} x^2-x+1, & x\leq 1 \\ a\sqrt{x+3}, & x>1 \end{cases}$

(D) $f(x)=\begin{cases} x^3+x+1, & x\geq 1 \\ a\sqrt{x+3}, & x<1 \end{cases}$

3D CONTINUED

Check at $x=4$

$\lim_{x\to 4^-}(-5) = -5$

$\lim_{x\to 4^+}(-2x+3) = -5$

$f(4) = -5 = -5$

$\therefore f(x)$ continuous AT $x=4$

Oct 14-11:31 AM

$f(x) = \frac{x}{x^2-x} = \frac{1}{x-1}$

at $x=1$

$\lim_{x\to 1^-}\left(\frac{1}{x-1}\right)$ * $\frac{k}{0}$

$= \frac{+}{-}$

$= -\infty$

$\lim_{x\to 1^+}\frac{1}{x-1}$ * $\frac{k}{0}$

$= \frac{+}{+}$

$= +\infty$

$\therefore x=1$ IS A V.A

HENCE Discontinuous (Infinite)

Oct 14-11:36 AM

* When Evaluating $f(a)$ use original

When Evaluating limits use simplified

Oct 14-11:53 AM

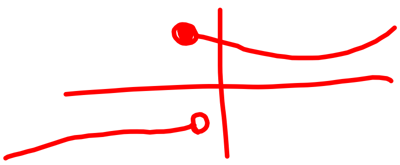
$$f(x) = \frac{|x+2|}{x+2}$$

$$\therefore \ |x+2| = \begin{cases} (x+2) & \text{if } x \geq -2 \\ -(x+2) & \text{if } x < -2 \end{cases}$$

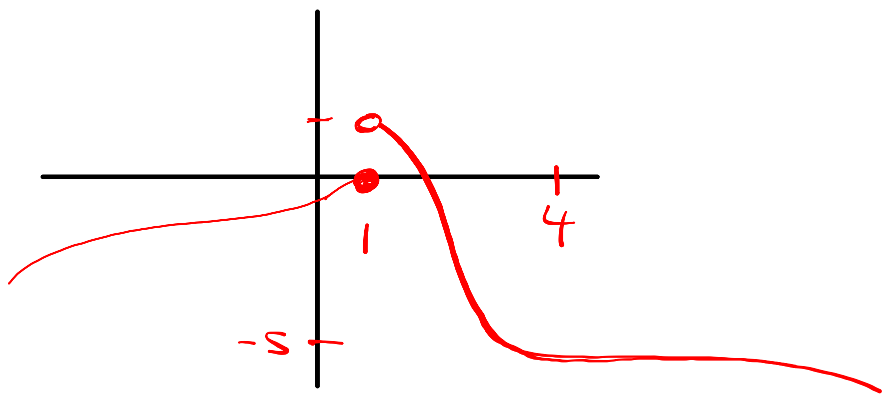
$$\lim_{x \to -2^+} \frac{(x+2)}{(x+2)} = 1$$

$$\lim_{x \to -2^-} \frac{-(x+2)}{(x+2)} = -1$$

$$f(-2) = \frac{\cancel{(x+2)}}{\cancel{x+2}} = 1$$



Oct 14-12:03 PM



Oct 14-1:22 PM

when check for continuity

→ check domain values
for piecewise functions

→ check values that make den = 0

Oct 14-1:33 PM

Oct 14-3:01 PM