Limits and Absloute Value.

Recall: $|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$ or $|x-a| = \begin{cases} x-a, & x \ge a \\ -(x-a), & x < a \end{cases}$

Evaluate each limit

1. $\lim\limits_{x \to 2^+} \frac{x^2-4}{|x-2|}$

$x-2 \ge 0$

$(x-2), \quad x \ge 2$

$-(x-2), \quad x < 2$

$\therefore \lim\limits_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = 4$

2. $\lim\limits_{x \to 3^-} \frac{x^2+2x-\cancel{10}\,15}{|x-3|}$

$x-3 \ge 0$

$|x-3| = \begin{cases} (x-3), & x \ge 3 \\ -(x-3), & x < 3 \end{cases}$

$\lim\limits_{x \to 3^-} \frac{\cancel{(x-3)}(x+5)}{-\cancel{(x-3)}}$

$= \lim\limits_{x \to 3^-} \frac{x+5}{-1} = -8$

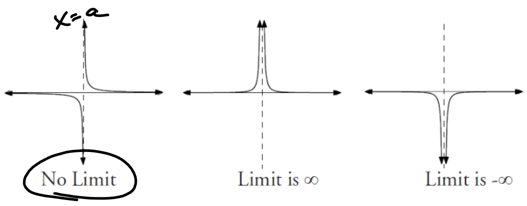
3. $\lim\limits_{x \to -1^+} \frac{x^3+1}{|x+1|}$

$\lim\limits_{x \to -1^+} \frac{\cancel{(x+1)}(x^2-1x+1)}{\cancel{(x+1)}}$ $\quad \begin{cases} (x+1) & x \ge -1 \\ -(x+1) & x < -1 \end{cases}$

$= (-1)^2 - 1(-1) + 1$

$= 1 + 1 + 1 = 3$

When when determining $\lim\limits_{x \to a} \frac{f(x)}{g(x)}$ and direct substitution yields $\frac{k}{0}$, where $k \ne 0$, this will yield one of the following situations.



Consider the following. Remember takes limit from both left and right.

a. $\lim\limits_{x \to 0} \frac{1}{x}$

b. $\lim\limits_{x \to 0} \frac{1}{x^2}$

c. $\lim\limits_{x \to 2} \frac{x+4}{x-2}$

$\lim\limits_{x \to 2^+}$

- as x approaches 2 from the left, the numerator approaches 6, and the denominator approaches 0 through negative values: $\lim_{x\to 2^-}\frac{x+4}{x-2}=-\infty$
- as x approaches 2 from the right, the numerator approaches 6, and the denominator approaches 0 through positive values: $\lim_{x\to 2^+}\frac{x+4}{x-2}=\infty$
- the $\lim_{x\to 2}\frac{x+4}{x-2}$ does not exist
- the function has a vertical asymptote at $x = 2$

You try:

1. $\lim_{x\to 3}\frac{x+1}{x-3}$

$\lim_{x\to 3^+}\frac{x+1}{x-3}=\frac{+}{+}=+\infty$

$\lim_{x\to 3^-}\frac{x+1}{x-3}=\frac{+}{-}=-\infty$

$\therefore \lim_{x\to 3}\frac{x+1}{x-3}=$ DNE

2. $\lim_{x\to -1}\frac{x}{(x+1)^2}$

$\lim_{x\to -1^+}\frac{x}{(x+1)^2}=\frac{-}{+}=-\infty$

$\lim_{x\to -1^-}\frac{x}{(x+1)^2}=\frac{-}{+}=-\infty$

$\therefore \lim_{x\to -1}\frac{x}{(x+1)^2}=-\infty$

3. $\lim_{x\to 5}\frac{3x}{x-5}$

$\lim_{x\to 5^+}\frac{3x}{x-5}=\frac{+}{+}=+\infty$

$\lim_{x\to 5^-}\frac{3x}{x-5}=\frac{+}{-}=-\infty$

$\therefore \lim_{x\to 5}\frac{3x}{x-5}=$ DNE

4. $\lim_{x\to 0}\frac{x-5}{x^4}$

$\lim_{x\to 0^+}\frac{x-5}{x^4}=\frac{-}{+}=-\infty$

$\lim_{x\to 0^-}\frac{x-5}{x^4}=\frac{-}{+}=-\infty$

$\therefore \lim_{x\to 0}\frac{x-5}{x^4}=-\infty$

Quiz Tomorrow

→ 9 Limits

- Graph
- Properties of limits
- Direct Sub
- Factor
- Com. Den
- Conjugate
- ABS. Value
- $\frac{K}{0}$

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$$\lim_{x \to 2} f(x) \qquad \therefore \ \underline{\underline{\text{dne}}}$$

$$f(x) = \begin{cases} x+1 & , x \geq 2 \\ 3x & , x < 2 \end{cases}$$

$$\lim_{x \to 2^+} (x+1) \Rightarrow 3$$

$$\lim_{x \to 2^-} (3x) = 6$$

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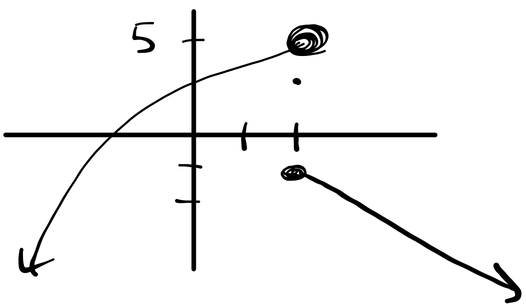

1. $\lim_{x \to 2^+} f(x)$ $\underline{-1}$

2. $\lim_{x \to 2^-} f(x)$ $\underline{5}$

3. $\lim_{x \to 2} f(x)$

Oct 1-11:34 AM