Solutions

Area Problems:

Example 1:

(a) $A(x) = -2x^2 + 60x$

$$x = \frac{-b}{2a} = \frac{-60}{2(-2)} = \frac{60}{4} = 15$$

$$A(15) = -2(15)^2 + 60(15) = -450 + 900 = 450$$

The maximum area is $450 m^2$

(b) Domain: $x \in \mathbb{R}$

Range: $\{y \mid y \leq 450\}$

Your Turn:

(a) $A(x) = -4x^2 + 120x$

$$x = \frac{-b}{2a} = \frac{-120}{2(-4)} = \frac{120}{8} = 15$$

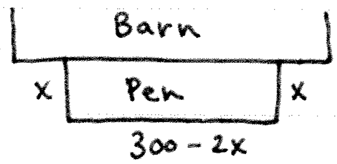
$$A(15) = -4(15)^2 + 120(15) = -900 + 1800 = 900$$

The maximum area is $900 m^2$

(b) Domain: $x \in \mathbb{R}$

Range: $\{y \mid y \leq 900\}$

Example 2:



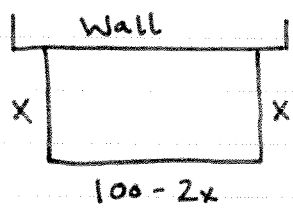
$$A(x) = (300-2x)(x)$$
$$= -2x^2 + 300x$$

$$x = \frac{-b}{2a} = \frac{-300}{2(-2)} = \frac{300}{4} = 75$$

$$A(75) = -2(75)^2 + 300(75) = -11250 + 22500 = 11250$$

The maximum area is $11250 m^2$

Your turn:



$$A(x) = (100-2x)(x)$$
$$= -2x^2 + 100x$$

$$x = \frac{-b}{2a} = \frac{-100}{2(-2)} = \frac{100}{4} = 25$$

$$A(25) = -2(25)^2 + 100(25) = -1250 + 2500 = 1250$$

The maximum area is $1250 m^2$

Revenue Problems:

Example 1:

(a) Number Sold = $(600-50x)$

Cost = $(50+5x)$

$$\begin{aligned} \text{Revenue} &= (\text{Number Sold})(\text{cost}) \\ &= (600-50x)(50+5x) \\ &= -250x^2 + 500x + 30000 \end{aligned}$$

(b) $x = \dfrac{-b}{2a} = \dfrac{-500}{2(250)} = \dfrac{-500}{-500} = 1$

$$\begin{aligned} \text{Revenue (max)} &= -250(1)^2 + 500(1) + 30000 \\ &= -250 + 500 + 30000 \\ &= 30250 \end{aligned}$$

$\therefore$ \$30 250.

(c)
$$\begin{aligned} \text{Cost (monthly fee)} &= 50 + 5(1) \\ &= 50+5 \\ &= 55 \end{aligned}$$

$\therefore$ \$55 per month.

Your Turn:

(a) Number sold = $(400 + 20x)$

Cost $= (60 - 2x)$

$$\begin{aligned}\text{Revenue} &= (\text{Number Sold})(\text{cost}) \\ &= (400+20x)(60-2x) \\ &= 24000 - 800x + 1200x - 40x^2 \\ &= -40x^2 + 400x + 24000\end{aligned}$$

(b) $x = \frac{-b}{2a} = \frac{-400}{2(-40)} = \frac{-400}{-80} = 5$

$$\begin{aligned}\text{Revenue (max)} &= -40(5)^2 + 400(5) + 24000 \\ &= -1000 + 2000 + 24000 \\ &= 25000\end{aligned}$$

$\therefore$ \$25000.

(c) Cost (price per crate) $= 60 - 2(5) = 60 - 10 = 50$

$\therefore$ \$50 per crate.

Extra Practice Problems:

1. Tickets Sold $= (400 - 20x)$
   Cost $= (10 + 2x)$

Revenue = (Tickets Sold)(cost)

$= ((400) - 20x)(10 + 2x)$

$= -40x^2 + 600x + 4000$

$$x = \frac{-b}{2a} = \frac{-600}{2(-40)} = \frac{-600}{-80} = \frac{15}{2}$$

Revenue (max) $= -40(15/2)^2 + 600(15/2) + 4000$

$= -2250 + 4500 + 4000$

$= 6250$

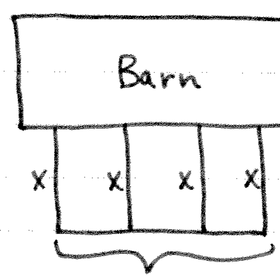
$\therefore$ \$6250 is the max revenue.

Cost (per ticket) $= 10 + 2(15/2)$

$= 10 + 15$

$= 25$ $\quad\therefore$ \$25 per ticket.

2.
(a)



$A(x) = -4x^2 + 240x$

$$x = \frac{-b}{2a} = \frac{-240}{2(-4)} = \frac{-240}{-8} = 30$$

$A(30) = -4(30)^2 + 240(30) = -3600 + 7200 = 3600$

$\therefore$ The maximum area is $3600\,m^2$

(b) Domain: $x \in \mathbb{R}$, Range: $\{y \mid y \leq 3600\}$

3. Tickets Sold = $(500 - 20x)$
Cost = $(100 + 5x)$

$$\begin{aligned}\text{Revenue} &= (\text{Tickets sold})(\text{cost}) \\ &= (500-20x)(100+5x) \\ &= 50000 + 2500x - 2000x - 100x^2 \\ &= -100x^2 + 500x + 50000\end{aligned}$$

$$X = \frac{-b}{2a} = \frac{-500}{2(-100)} = \frac{-500}{-200} = \frac{5}{2}$$

$$\begin{aligned}\text{Revenue (max)} &= (-100)(5/2)^2 + 500(5/2)^2 + 50000 \\ &= -625 + 3125 + 50000 \\ &= 52500\end{aligned}$$

$\therefore$ \$52500 is the max revenue.

$$\begin{aligned}\text{Cost (per ticket)} &= 100 + 5(5/2) \\ &= 100 + 12.5 \\ &= 112.5\end{aligned}$$

$\therefore$ \$112.50 is the price per ticket.

4. (a) $A(x) = -x^2 + 60x$

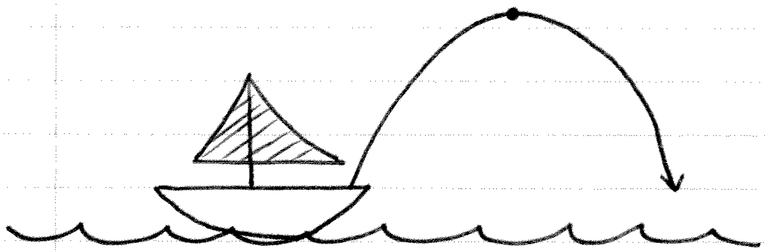
$$X = \frac{-b}{2a} = \frac{-60}{2(-1)} = \frac{-60}{-2} = 30$$

$$A(30) = -(30)^2 + 60(30) = -900 + 1800 = 900$$

$\therefore$ The maximum area is $900\,m^2$.

(b) Domain: $x \in \mathbb{R}$, Range: $\{y \mid y \leq 900\}$.

5. $h(t) = -4.9t^2 + 29.4t + 3$



$x = \frac{-29.4}{2(-4.9)} = \frac{-29.4}{-9.8} = 3$ $\therefore$ 3 seconds to reach max height.

$h(3) = -4.9(3)^2 + 29.4(3) + 3$
$= -44.1 + 88.2 + 3$
$= 47.1$

$\therefore$ The maximum height is 47.1 m.

6. $h(t) = -\frac{1}{2}t^2 + 5t + 2$

$x = \frac{-b}{2a} = \frac{-5}{2(-\frac{1}{2})} = \frac{-5}{-1} = 5$ $\therefore$ maximum height is reached after 5 seconds.

$h(5) = -\frac{1}{2}(5)^2 + 5(5) + 2$

$= -\frac{1}{2}(25) + (25) + 2$

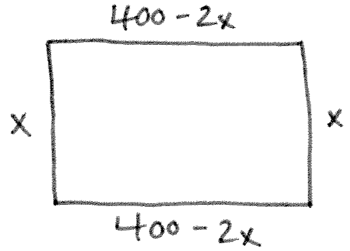
$= -12.5 + 25 + 2$

$= 14.5$

(We only need to find the time here)

$\therefore$ The maximum height is 14.5 m.

7.



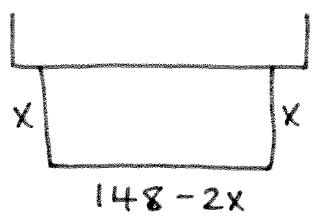
$A(x) = (400-2x)(x) = -2x^2 + 400x$

$x = \frac{-b}{2a} = \frac{-400}{2(-2)} = \frac{-400}{-4} = 100$

$A(100) = -2(100)^2 + 400(100)$
$= -20000 + 40000$
$= 20000$

$\therefore$ the maximum area is $20000\,m^2$.

8.



$A(x) = (148-2x)(x)$
$= -2x^2 + 148x$

$x = \frac{-b}{2a} = \frac{-148}{2(-2)} = \frac{-148}{-4} = 37$

$A(37) = -2(37)^2 + 148(37)$
$= -2738 + 5476$
$= 2738$

$\therefore$ The maximum area is $2738\,m^2$.