

Key



**Mathematics 2201
Common Mathematics Assessment**

June 12, 2013

Name:

Mathematics

Teacher:

28 Selected Response

13 Constructed Response

28 marks

42 marks

FINAL

70 Marks

TIME: 2 HOURS

NOTE

Diagrams are not necessarily drawn to scale.

FORMULAE

| | | |
|--|--------------------------------|--|
| $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ | $a^2 = b^2 + c^2 - 2bc \cos A$ | $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ |
|--|--------------------------------|--|

| | | |
|---|------------------------------|--|
| $\sigma = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}$ | $z = \frac{x - \mu}{\sigma}$ | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
|---|------------------------------|--|

Selected Response: Choose the appropriate response on the answer sheet or SCANTRON.

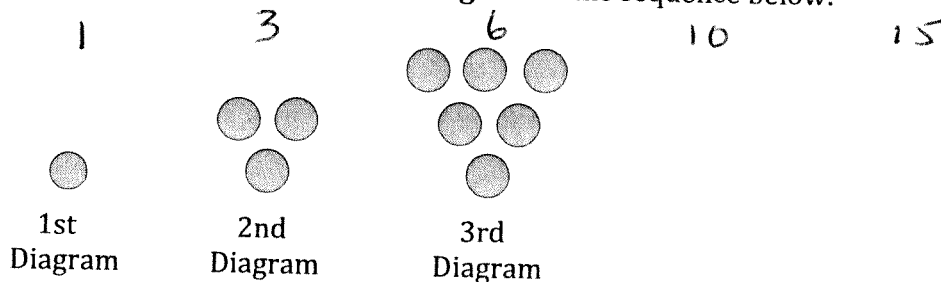
1. What is a statement that is believed to be true but not yet proven?

- (A) Conjecture
- (B) Counterexample
- (C) Deductive Reasoning
- (D) Inductive Reasoning

2. Which is a counterexample to the statement "The sum of two consecutive integers is always greater than each of the two integers"?

- (A) $-4 + (-5) = -9$
- (B) $4 + (-5) = -1$
- (C) $-4 + 5 = 1$
- (D) $4 + 5 = 9$

3. How many circles are in the 5th diagram in the sequence below:

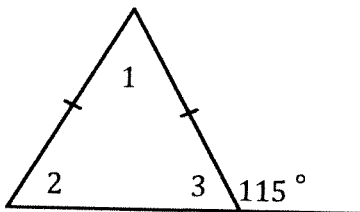


- (A) 9
- (B) 10
- (C) 14
- (D) 15

4. If two non-parallel lines are cut by a transversal, which pair of angles is always equal?

- (A) Alternate Interior
- (B) Corresponding
- (C) Supplementary
- (D) Vertically Opposite

5. A student was asked to find the measure of $\angle 1$. In which step did he make the first error?



Solution

Step 1: $\angle 3 = 180^\circ - 115^\circ$ ✓

Step 2: $\angle 3 = 65^\circ$ ✓

Step 3: $\angle 1 = \angle 3$ ✗

Step 4: $\angle 1 = 65^\circ$

- (A) 1
- (B) 2
- (C) 3
- (D) 4

6. How many sides does a convex polygon have if the sum of its interior angles is 1440° ?

- (A) 4
(B) 6
(C) 8
(D) 10

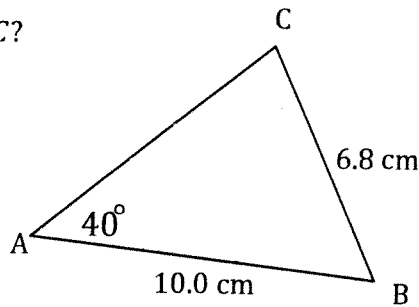
$$\frac{1440}{180} = \frac{180(n-2)}{180}$$

$$8 = n - 2$$

$$10 = n$$

7. What is the measure of $\angle C$?

- (A) 20°
(B) 26°
(C) 69°
(D) 71°

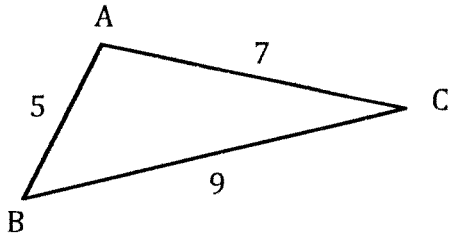


$$\frac{\sin 40^\circ}{6.8} = \frac{\sin C}{10}$$

$$0.9453 = \sin C$$

$$71^\circ = C$$

8. Which equals the measure of $\angle A$?



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{7^2 + 5^2 - 9^2}{2(7)(5)}$$

- (A) $\cos^{-1}\left(\frac{5^2 + 9^2 - 7^2}{2(5)(9)}\right)$
(B) $\cos^{-1}\left(\frac{7^2 + 5^2 - 9^2}{2(7)(5)}\right)$
(C) $\cos^{-1}\left(\frac{9^2 + 5^2 - 7^2}{2(9)(5)}\right)$
(D) $\cos^{-1}\left(\frac{9^2 + 7^2 - 5^2}{2(9)(7)}\right)$

9. Simplify completely:

$$5\sqrt{7} + 3\sqrt{28}$$

- (A) $11\sqrt{7}$
(B) $17\sqrt{7}$
(C) $11\sqrt{14}$
(D) $8\sqrt{35}$

$$5\sqrt{7} + 3\sqrt{4 \cdot 7}$$

$$5\sqrt{7} + 6\sqrt{7}$$

$$11\sqrt{7}$$

10. Simplify completely:

$$\sqrt[3]{-8x^{17}}$$

- (A) $-2x^2 \sqrt[3]{x^5}$
(B) $-2x^5 \sqrt[3]{x^2}$
(C) $2x \sqrt[3]{-2x^8}$
(D) $2x^8 \sqrt[3]{-2x}$

$$\sqrt[3]{-8x^{15} \cdot x^2}$$

$$-2x^5 \sqrt[3]{x^2}$$

11. Write $3x^3\sqrt{5x}$ as an entire radical.

- (A) $\sqrt{15x^4}$
 (B) $\sqrt{15x^7}$
 (C) $\sqrt{45x^4}$
 (D) $\sqrt{45x^7}$

$$\sqrt{3^2 x^6 \cdot 5x} = \sqrt{45x^7}$$

12. A student was asked to simplify $\frac{x\sqrt{18x^3}}{3}$ but did not complete a correct solution. Which step contains her first error?

Solution: Step 1: $\frac{x\sqrt{9 \cdot 2 \cdot x^2 \cdot x}}{3}$

Step 2: $\frac{x \cdot 9x^2 \sqrt{2x}}{3}$

Step 3: $\frac{9x^3 \sqrt{2x}}{3}$

Step 4: $3x^3 \sqrt{2x}$

- (A) 1
 (B) 2
 (C) 3
 (D) 4

13. Simplify completely:

(A) $\frac{10\sqrt{6}}{3}$

(B) $\frac{40\sqrt{6}}{3}$

(C) $\frac{5\sqrt{96}}{6}$

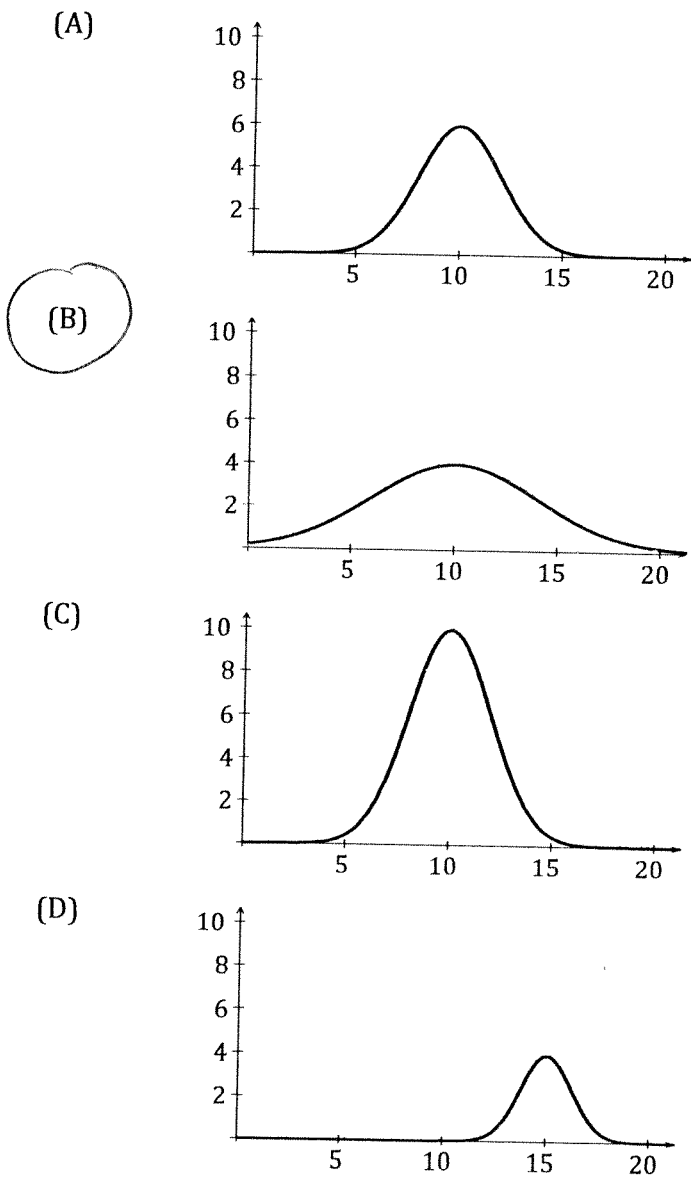
(D) $\frac{10\sqrt{96}}{12}$

$$\frac{5\sqrt{32}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{16 \cdot 2 \cdot 3}}{2 \cdot 3} = \frac{20\sqrt{6}}{6} = \frac{10\sqrt{6}}{3}$$

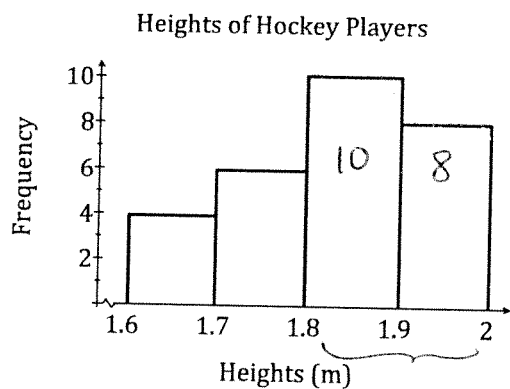
14. What are the restrictions on the variable for $\sqrt{x+2}$?

- (A) $x \geq -2$
 (B) $x > -2$
 (C) $x \geq 2$
 (D) $x > 2$

15. Which represents data with the largest standard deviation?



16. The histogram shown represents the heights of hockey players on a professional hockey team. How many players have a height between 1.8 m and 2.0 m?



- (A) 10
- (B) 18
- (C) 24
- (D) 28

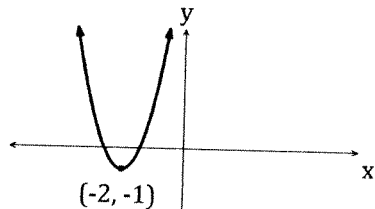
17. A set of data is normally distributed. What percent of the data is within two standard deviations of the mean?

- (A) 47.5
- (B) 68
- (C) 95
- (D) 99.7



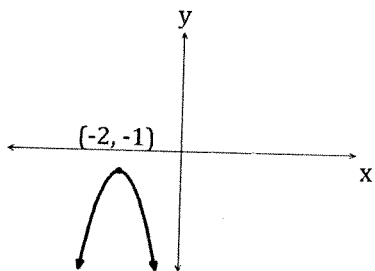
18. The function $y = -3x^2 - 12x - 13$ has axis of symmetry $x = -2$. Which represents the function?

(A)

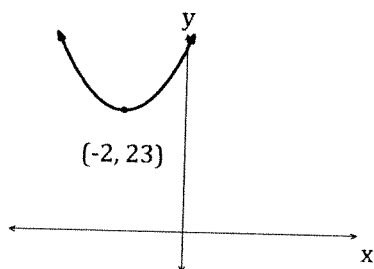


$$\begin{aligned}
 y &= -3(-2)^2 - 12(-2) - 13 \\
 &= -12 + 24 - 13 \\
 &= 12 - 13 \\
 &= -1
 \end{aligned}$$

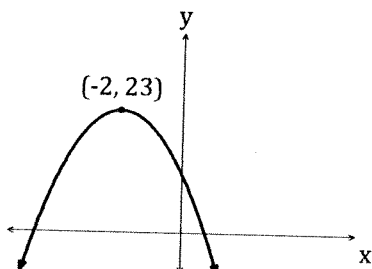
(B)



(C)



(D)



19. What is the domain and range for $f(x) = -2(x + 1)^2 - 3$?

- (A) $x \in \mathbb{R}$ and $f(x) \leq -3$
- (B) $x \in \mathbb{R}$ and $f(x) \geq -3$
- (C) $x \leq -1$ and $f(x) \in \mathbb{R}$
- (D) $x \geq -1$ and $f(x) \in \mathbb{R}$

20. A parabola has x-intercepts of $(-2,0)$ and $(-8,0)$. What is the axis of symmetry?

- (A) $x = -5$
 (B) $x = -3$
 (C) $y = -5$
 (D) $y = -3$



21. What is the vertex of $y = 2x^2 + 8x - 5$?

- (A) $(-2, -29)$
 (B) $(-2, -13)$
 (C) $(2, 15)$
 (D) $(2, 19)$

$$V_x = \frac{-8}{2(2)} = \frac{-8}{4} = -2$$

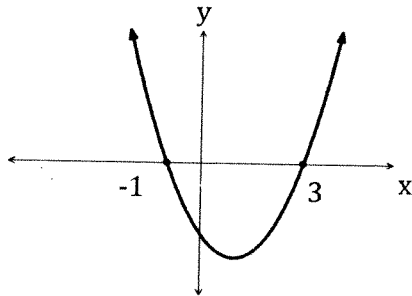
$$V_y = 2(-2)^2 + 8(-2) - 5 = 8 - 16 - 5 = -8 - 5 = -13$$

22. The graph of a quadratic function has vertex $(1, -4)$ and opens upward. How many x-intercepts does it have?

- (A) 0
 (B) 1
 (C) 2
 (D) 3



23. What is the equation of the function graphed below?



- (A) $y = (x - 1)(x - 3)$
 (B) $y = (x - 1)(x + 3)$
 (C) $y = (x + 1)(x - 3)$
 (D) $y = (x + 1)(x + 3)$

24. Which is a root of $2x^2 - 5x - 3 = 0$

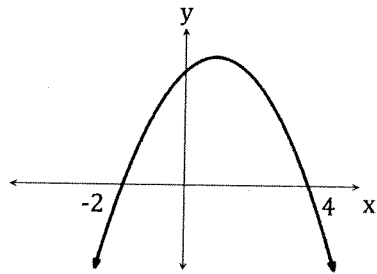
- (A) -3
 (B) -1
 (C) 1
 (D) 3

$$(2x+1)(x-3) = 0$$

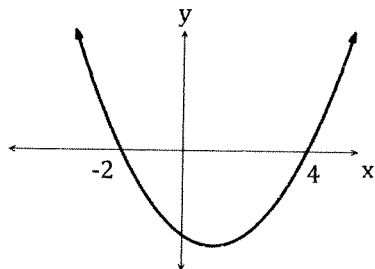
$$x = 3$$

25. Which represents a quadratic function with zeros of -2 and 4 and a maximum value?

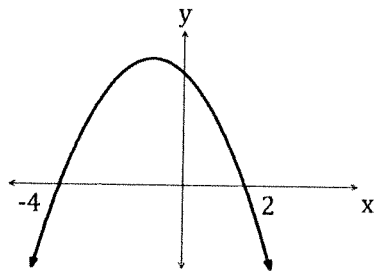
(A)



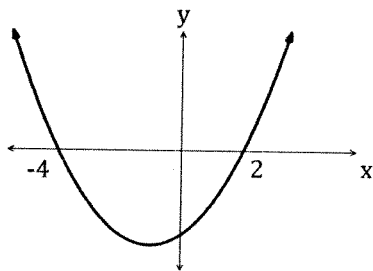
(B)



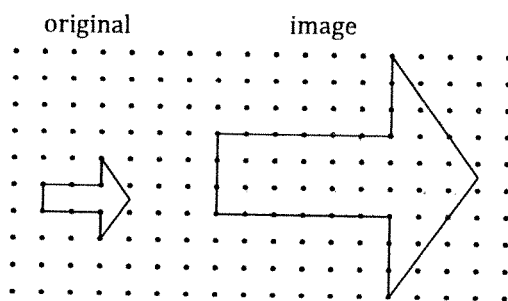
(C)



(D)



26. What is the scale factor in the figure below?



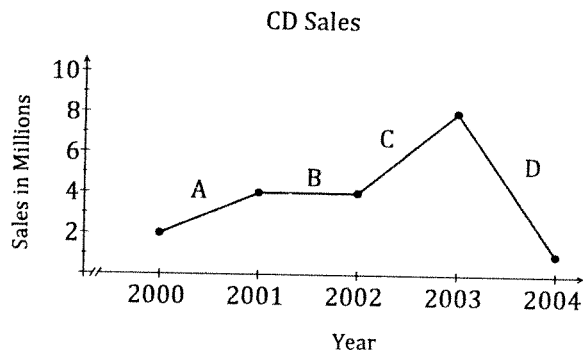
(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) 2

(D) 3

27. During which time period was the growth rate of CD sales the greatest in the graph shown?



- (A) 2000 - 2001
(B) 2001 - 2002
(C) 2002 - 2003
(D) 2003 - 2004
28. The surface area of a cone is 34 ft^2 . If the cone is enlarged by a scale factor of 3, what is the surface area, in ft^2 , of the image?

- (A) 37
(B) 102
(C) 306
(D) 918

$$34 \cdot 9$$

Constructed Response:

Answers to be written on this paper in the space provided. Show all workings.

29. Use **both** inductive and deductive reasoning to show that the result for the given [4 marks] number trick will always be the original number.

NUMBER TRICK

Choose a number.

Double it.

Add 6.

Double it

Subtract 4.

Divide by 4.

Subtract 2.

Inductive Reasoning

ex. 3 ? ?

6

12

24

20

5

3

①

?

Deductive Reasoning

n

2n

2n+6

4n+12

4n+8

n+2

n

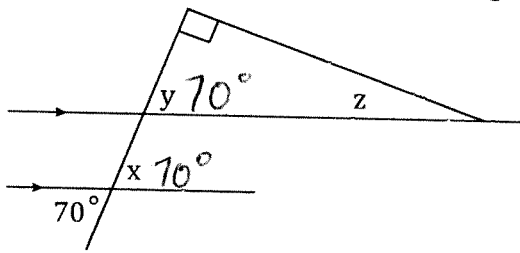
③

?

$\left. \begin{array}{l} \checkmark \\ \checkmark \\ \checkmark \\ \checkmark \\ \checkmark \\ \checkmark \end{array} \right\} \left(\frac{1}{2}\right) \text{ each.}$

30. Find the measure of each indicated angle. Justify your answer.

[3 marks]



| Angle Measure | Justification |
|----------------|---|
| $x = 70^\circ$ | vertically opposite \angle corresponding \angle - parallel lines angle sum for $\Delta = 180^\circ$ |
| $y = 70^\circ$ | |
| $z = 20^\circ$ | |

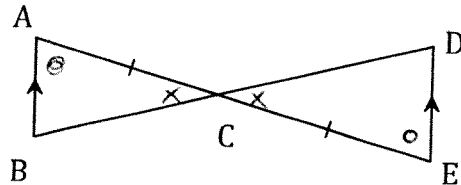
(Handwritten note: 1/2 each with arrows pointing to the angle measures)

31. Use either a paragraph or two-column format to complete the given proof:

[3marks]

Given: $AB \parallel DE$
 $AC = EC$

Prove: $\Delta ABC \cong \Delta EDC$

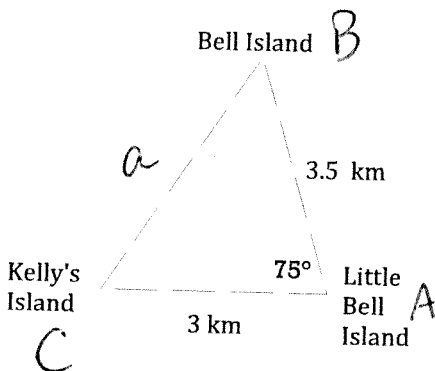


| S | R |
|--|---|
| ? 1. $AB \parallel DE$ ① 2. $\angle A = \angle E$ 3. $AC = EC$ ① 4. $\angle ACB = \angle ECD$ ① 5. $\Delta ABC \cong \Delta EDC$ | 1. Given 2. Parallel lines/ alt. int. \angle 3. Given 4. Vert. opp. \angle 5. ASA. |

OR ... ?

32. A boat travels from Bell Island to Kelly's Island to Little Bell Island, and returns directly back to Bell Island. What is the total distance travelled?

[4 marks]



$$\begin{aligned}
 & \frac{1}{2} a^2 = b^2 + c^2 - 2bc \cos A \\
 & \textcircled{1} a^2 = 3^2 + 3.5^2 - 2(3)(3.5)\cos 75^\circ \\
 & \textcircled{1} \begin{cases} a^2 = 21.25 - 5.4352 \\ a^2 = 15.8148 \end{cases} \\
 & \textcircled{2} a = \sqrt{15.8148} \\
 & \textcircled{2} a = 3.98 \text{ or } 4.0
 \end{aligned}$$

Total Distance
 $= 3 + 3.5 + 4.0$ $\textcircled{2}$
 $= 10.5 \text{ Km}$

33. Simplify completely: $5\sqrt{6}(\sqrt{3} + 3\sqrt{12} - \sqrt{2})$ [3 marks]

$$\begin{aligned}
 &= 5\sqrt{18} + 15\sqrt{72} - 5\sqrt{12} \quad (1) \\
 &= 5\sqrt{9 \cdot 2} + 15\sqrt{36 \cdot 2} - 5\sqrt{4 \cdot 3} \quad (1) \\
 &= 5 \cdot 3\sqrt{2} + 15 \cdot 6\sqrt{2} - 5 \cdot 2\sqrt{3} \\
 &= 15\sqrt{2} + 90\sqrt{2} - 10\sqrt{3} \quad (2) \\
 &= 105\sqrt{2} - 10\sqrt{3} \quad (2)
 \end{aligned}$$

34. State the **restrictions** on x , **solve** the equation, and then **check** for extraneous roots. [4 marks]

$$\begin{aligned}
 &\sqrt{3x+1} - 3 = -4 \quad \text{Restrictions} \quad \begin{aligned} &3x+1 \geq 0 \\ &3x \geq -1 \\ &x \geq -\frac{1}{3} \end{aligned} \quad (1) \\
 &\sqrt{3x+1} = -4 + 3 \\
 &\sqrt{3x+1} = -1 \quad (2) \\
 &(\sqrt{3x+1})^2 = (-1)^2 \\
 &3x+1 = 1 \\
 &3x = 0 \\
 &x = 0
 \end{aligned}$$

Check:
 $\sqrt{3(0)+1} - 3 \quad (1)$
 $= \sqrt{1} - 3$
 $= 1 - 3$
 $= -2 \neq -4$ no solution.

35. A factory produces automotive brake pads with a mean mass of 174 g and a standard deviation of 0.7 g. Quality control expects that the mass of the pads will lie within the acceptable range of 173.9 g and 174.1 g. What is the confidence interval and margin of error this factory uses for its quality control tests? [2 marks]

Confidence Interval: $173.9 - 174.1 \quad (1)$

$$\begin{aligned}
 173.9 - 174 &= -0.1 \quad (1/2) \\
 174.1 - 174 &= 0.1 \quad (1/2)
 \end{aligned}$$

Margin of error is $\pm 0.1 \quad (1/2)$

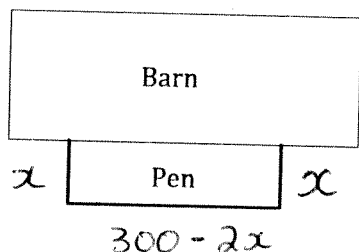
36. Jason scored 82% on a test where the class average was 74% and the standard deviation was 10.6%. If the class was normally distributed, what percentage of the class scored better than Jason? [3 marks]

$$\textcircled{1} \quad z\text{-score} = \frac{x - \mu}{\sigma} = \frac{82 - 74}{10.6} = 0.75$$

$\textcircled{1}$ \rightarrow from table we get : 0.7734 or 77.34% which is below the mean (or equal to).

$\textcircled{1}$ $\therefore 100\% - 77.34\% = 22.66\%$ scored better than Jason.

37. A farmer has 300 m of chain link fencing to create a rectangular pen, using the side of a barn as one side of the pen. Algebraically determine the maximum area that can be enclosed by the pen. [4 marks]



$$\begin{aligned} x &= \text{width} \\ 300 - 2x &= \text{length} \end{aligned} \quad \textcircled{1}$$

$$\begin{aligned} \text{Area} &= x(300 - 2x) \\ A &= 300x - 2x^2 \end{aligned} \quad \textcircled{1}$$

max. at vertex $\Rightarrow V_x = \frac{-b}{2a} = \frac{-300}{2(-2)} = 75 \quad \textcircled{1}$

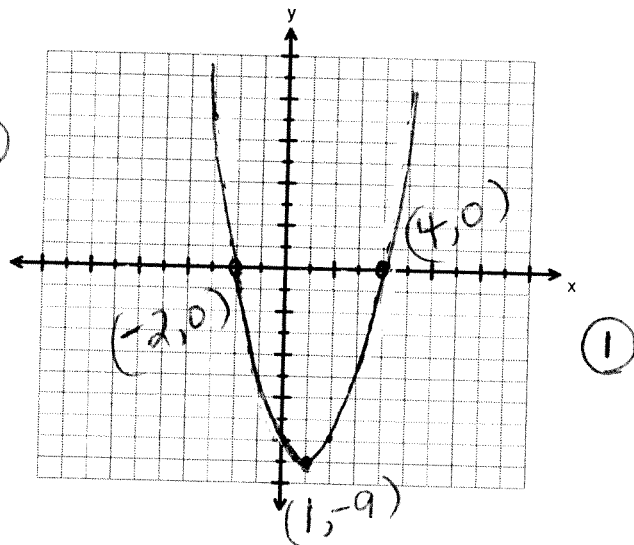
$$\begin{aligned} \text{max. area} &= 75(300 - 2(75)) & \text{or } A &= 300(75) - 2(75)^2 \\ \textcircled{1} &= 75(150) & &= 11250. \\ &= 11250 \text{ m}^2. & & \end{aligned}$$

38. Algebraically determine the **vertex** and **x-intercepts** for the function $y = x^2 - 2x - 8$. Sketch the graph, labelling all key points. [3 marks]

$$V_x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1$$

$$\begin{aligned} V_y &= 1^2 - 2(1) - 8 \\ &= 1 - 2 - 8 \\ &= -9 \end{aligned} \quad \textcircled{1} \quad (1, -9)$$

x-int: $0 = x^2 - 2x - 8$
 $0 = (x - 4)(x + 2)$
 $x = 4 \text{ or } x = -2$



39. Solve the given equation. State the solution(s) in exact form.

[3 marks]

$$6x^2 = -4x + 3$$

$$\left(\frac{1}{2}\right) 6x^2 + 4x - 3 = 0$$

$$x = \frac{-2 \pm \sqrt{22}}{6}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(6)(-3)}}{2(6)} \left(\frac{1}{2}\right)$$

$$x = \frac{-4 \pm \sqrt{16 + 72}}{12}$$

$$x = \frac{-4 \pm \sqrt{88}}{12}$$

$$x = \frac{-4 \pm \sqrt{4 \cdot 22}}{12} = \frac{-4 \pm 2\sqrt{22}}{12}$$

$$= \frac{-2 \pm \sqrt{22}}{6}$$

$$\uparrow \left(\frac{1}{2}\right)$$

[3 marks]

40. On another planet, the path of a rock that is thrown is given by $h = -t^2 + 4t + 6$, where h is height in metres and t is time in seconds. At what time(s) would the height of the ball be 9 m?

$$\textcircled{1} 9 = -t^2 + 4t + 6$$

$$t^2 - 4t - 6 + 9 = 0$$

$$\left(\frac{1}{2}\right) t^2 - 4t + 3 = 0$$

$$\left(\frac{1}{2}\right) \left\{ \begin{array}{l} (t-3)(t-1) = 0 \\ t=3 \quad t=1 \end{array} \right\} \text{ or use Q.F.}$$

At $t=1$ and $t=3$ seconds the rock would be 9m high.

41. Avalon Supermarket sells a box of 48 granola bars for \$7.99 and a box of 8 bars for \$1.99. What is the least expensive way to buy 70 granola bars? Justify your reasoning. *

[3 marks]

$$\#1 \quad 2 \text{ boxes of } 48 = 2(7.99) = \$15.98 \text{ for } 96 \text{ bars} \quad \textcircled{1}$$

(96 bars)

$$\#2 \quad 9 \text{ boxes of } 8 = 9(1.99) = \$17.91 \text{ for } 72 \text{ bars} \quad \textcircled{1}$$

(72 bars)

$$\#3 \quad 1 \text{ box of } 48 + 3 \text{ boxes of } 8 = 7.99 + 3(1.99) \quad \textcircled{1}$$

(48) \quad (24) \quad = \\$13.96 \text{ for } 72 \text{ bars.}

The least expensive option is #3.

Mathematics 2201 Common Assessment - June 2013
Answer Sheet

Name: _____

Mathematics Teacher: _____

- | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1. | (A) | B | C | D | 15. | A | (B) | C | D |
| 2. | (A) | B | C | D | 16. | A | (B) | C | D |
| 3. | A | B | C | (D) | 17. | A | B | (C) | D |
| 4. | A | B | C | (D) | 18. | A | (B) | C | D |
| 5. | A | B | (C) | D | 19. | (A) | B | C | D |
| 6. | A | B | C | (D) | 20. | (A) | B | C | D |
| 7. | A | B | C | (D) | 21. | A | (B) | C | D |
| 8. | A | (B) | C | D | 22. | A | B | (C) | D |
| 9. | (A) | B | C | D | 23. | A | B | (C) | D |
| 10. | A | (B) | C | D | 24. | A | B | C | (D) |
| 11. | A | B | C | (D) | 25. | (A) | B | C | D |
| 12. | A | (B) | C | D | 26. | A | B | C | (D) |
| 13. | (A) | B | C | D | 27. | A | B | (C) | D |
| 14. | (A) | B | C | D | 28. | A | B | (C) | D |