MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAMINATION **Mathematics 1000** **WINTER 2011**

Marks

[12] 1. Using methods learned in this course, evaluate the following limits, showing your work.

a) $\lim\limits_{x\to -2} \dfrac{x^2-2x-8}{x^2+5x+6}$

b) $\lim\limits_{x\to 3} \dfrac{4-\sqrt{x^2+7}}{x^2-9}$

c) $\lim\limits_{x\to 7^+} \dfrac{|7-x|}{x^2-49}$

d) $\lim\limits_{x\to 0} \dfrac{\sin^2(5x)}{x^2\cos^2(2x)}$

[6] 2. Let $f(x) = \begin{cases} x+1, & \text{for } x < 1 \\ 2, & \text{for } 1 \le x \le 2 \\ x-1, & \text{for } x > 2 \end{cases}$.

Using the definition of continuity, determine all values at which $f(x)$ is discontinuous. Classify any discontinuities as removable or non-removable.

[7] 3. Use the DEFINITION OF DERIVATIVE to find $f'(x)$ for $f(x) = \sqrt{3x-4}$.

4. Differentiate each function and make any appropriate simplifications.

[4] a) $y = \tan^3\left(\sqrt{3x^2-2x}\right)$ DO NOT USE LOGARITHMIC DIFFERENTIATION

[3] b) $y = \dfrac{x^2-6}{(4x-3)^3}$ DO NOT USE LOGARITHMIC DIFFERENTIATION

[4] c) $y = (e^{x^2})(\cot 6x^3)$ DO NOT USE LOGARITHMIC DIFFERENTIATION

[6] d) $y = (\sin x)^{x^2}$

[6] 5. A closed box with a square base is to have a volume of 10 m^3.The base costs \$4/m^2, the sides cost \$2/m^2 and the top \$1/m^2. What dimensions will give the minimum cost to build the box?

[7] 6. Car A is being driven south toward point P at a speed of 60 km/h. Car B is being driven to the east away from point P. When car A is 0.6 km from point P, car B is 0.8 km from point P and the stright line distance between them is increasing at 20 km/h. What is the speed of car B?

7. Find each of the following integrals.

[3] a) $\int [\sin(3x-2) + e^{2x}]\,dx$

[3] b) $\int \left[\dfrac{(x+2)^2}{x}\right] dx$

[4] 8. a) Use implicit differentiation to find y' for $3x^2+2xy^2+y^3-19 = 0$.

[3] b) Use y' from part a) to find the equation of the tangent line at $(1, 2)$.

9. Given the following: $f(x) = \dfrac{x+1}{(x-1)^2}$, $f'(x) = \dfrac{-(x+3)}{(x-1)^3}$ and $f''(x) = \dfrac{2x+10}{(x-1)^4}$.

[3] a) Find the vertical asymptotes of $f(x)$, if any.

[3] b) Find the horizontal asymptotes of $f(x)$, if any.

[2] c) Find the x- and y-intercepts of the graph of $f(x)$, if any.

[3] d) Determine the intervals on which $f(x)$ is increasing or decreasing and classify any relative (local) extrema.

[3] e) Determine the intervals on which $f(x)$ is concave up or concave down and identify any inflection points.

[3] f) Sketch the graph of $f(x)$. Label any inflection points and extrema.

[7] 10. Find the area bounded by the graphs of $y = x^2 - 5x - 7$ and $y = 3x + 3 - x^2$.

[8] 11. Answer ONE of the following:

a) Prove that if $H(x) = f(x) - g(x)$ then $H'(x) = f'(x) - g'(x)$.

OR

b) Given that $xy + y^2 = 1$, show that $y'' = \dfrac{2}{(x+2y)^3}$.