MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAMINATION | **Mathematics 1000** | **FALL 2010**

Marks

[12] 1. Evaluate the following limits, showing your work. Assign ∞ or $-\infty$ as appropriate.

a) $\lim_{x\to 3} \frac{x^2+2x-15}{2x^2-5x-3}$ b) $\lim_{x\to -1} \frac{4x+4}{2x+\sqrt{x^2+3}}$ c) $\lim_{x\to 0} \frac{\sin^2(4x)}{x^2}$

[8] 2. Given the function $f(x) = \begin{cases} \frac{6}{1-x}, & \text{for } x \le -1 \\ \frac{x-2}{x^2-4}, & \text{for } x > -1 \end{cases}$, using the definition of continuity, determine all points at which $f(x)$ is not continuous. Classify any discontinuities as removable or non-removable.

[10] 3. a) State the limit definition of the derivative of a function $f(x)$.

b) Use the limit definition to differentiate $f(x) = \frac{3x+1}{2-x}$.

c) Verify your answer to part b) using the quotient rule.

[16] 4. Compute $\frac{dy}{dx}$, making any obvious simplifications.

a) $y = x^5 e^{x^5}$ b) $y = \ln\left(\frac{\sqrt{x}}{e^x \tan^3 x}\right)$ c) $y = \sin^7(\sqrt{x})$

d) $y = (\sec x)^{4x}$

[5] 5. Using Implicit differentiation, find the equation of the tangent line to the curve defined by the equation $\pi(x^2 - y^2) = \cos(\pi y)$ at the point $(3/2, 3/2)$.

[8] 6. An airplane is flying due east at a constant altitude of 16 km, and passes directly over an air control tower. Later, the plane passes over a route marker which lies 12 km east of the tower. At this moment, the distance between the tower and the plane is measured to be changing at a rate of 400 km per hour. What is the speed of the plane at this time?

[15] 7. Consider the function $f(x) = \frac{4x}{(x-1)^2}$ with derivatives $f'(x) = \frac{-4(x+1)}{(x-1)^3}$ and $f''(x) = \frac{8(x+2)}{(x-1)^4}$.

a) Find any vertical asymptotes of the graph of f.
b) Find any horizontal asymptotes of the graph of f.
c) Find any x- and y-intercepts of the graph of f.
d) Find the intervals on which $f(x)$is increasing or decreasing, and classify any relative (local) extrema.
e) Determine the intervals on which the graph of f is concave upward or concave downward and identify any points of inflection.
f) Sketch the graph of f. Label your graph carefully.

[12] 8. Find each of the following integrals.

a) $\displaystyle\int \frac{(x-2)^2}{x}\,dx$ b) $\displaystyle\int \sqrt{5x+4}\,dx$ c) $\displaystyle\int_0^{\pi} \cos\left(\frac{x}{6}\right)\,dx$

[6] 9. Indicate whether each of the following statements is true or false. If the statement is true, briefly explain why. If the statement is false, briefly explain why or give a counterexample that illustrates this.

a) If a function f is continuous at a point, then it is also differentiable at that point.

b) A function f defined on an open interval must achieve an absolute (global) maximum and an absolute (global) minimum value.

[8] 10. Do **EITHER** part a) **OR** part b).

a) A farmer wants to fence a rectangular plot of land with area 2400 m^2. She wants to keep her horses on one side of the field, so she plans to use additional fencingto build an internal divider, parallel to two sides of the fence. Wood for the outer walls costs \$3 per metre, and wood for the internal divider costs \$2 per metre. What is the minimum cost of the project?

b) Find the area of the region enclosed by the curves $y = 1 - x^2$ and $y = (x+1)(7-3x)$.