

**Confidence Intervals**

*Section 5.6*

Statistical inference provides methods for drawing conclusions about population from sample data. There are two major types of statistical inference: **Confidence intervals** and **tests of significance**. We will focus only on understanding and using confidence intervals.

**Margin of error** is the possible difference between the sample estimate and the true value for the population proportion. It is usually expressed as a plus or minus percent, such as  $\pm 4.5\%$ .

**Confidence interval** is the interval in which the true population proportion is estimated to lie, with a stated degree of probability, based on the results of the sample. Confidence interval is usually expressed using  $\pm$  notation, such as  $64\% \pm 4.5\%$  or ranging from  $59.5\%$  to  $68.5\%$ .

The range of values from  $59.5\%$  to  $68.5\%$  is called an **interval**.

**Confidence level** is the likelihood that the true population proportion lies within the range of the confidence interval.

A confidence interval of  $95\%$  means that if the survey was repeated over and over again,  $95\%$  of the time the results would fall within the confidence interval given.

68% of the data is  $\pm 1$  standard deviations away from the mean.

95% of the data is  $\pm 2$  standard deviations away from the mean.

99.7% of the data is  $\pm 3$  standard deviations away from the mean.

For the  $95\%$  interval, this means that in  $95\%$  of all samples of 100 students from this population, the mean score for the sample will fall within \_\_\_\_\_ standard deviations of the true population mean or \_\_\_\_\_ points from the mean.

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Unit 5: Statistics Lesson 6

We rarely know the population mean, but in practice, we take samples and create confidence intervals as a method of estimating the true value of the parameter.

When we find a  $95\%$  confidence interval, we believe with  $95\%$  confidence that the true parameter falls within our interval.

However, we must accept that  $5\%$  of all samples will give intervals which do not include the parameter.

Every confidence interval takes the same shape: **estimate  $\pm$  margin of error**.

A random sample of 100 teenagers was surveyed, and the mean number of DVD movies that they had rented in the past month was 9.4 with results considered accurate within 1.4, 18 times out of 20.



- What % of confidence level are the results?
- What is the margin of error?
- What is the confidence interval? Explain.

A hockey puck manufacturer quality control officer conducts sampling in order to determine the mean mass of the hockey pucks being produced. He reports that the mass of the hockey pucks produced at the factory is  $127.2$  grams  $\pm 0.4$  grams. the result is accurate 19 times out of 20.



- Determine the confidence level.
- Determine the confidence interval.
- State the margin of error.
- Is it likely that the mean mass of hockey pucks produced at this factory is  $126.5$  grams? Explain.
- Is it possible that the mean mass of hockey pucks produced at this factory is  $126.5$  grams? Explain.

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**The Effect of Sample Size on Confidence Level**


➡ In order to get 100% confident about the results of your measurements, you would need to measure the entire population.

➡ If, in our sample, we had measured **more than half** of a population such as students in a school, our **level of confidence** that the true mean lies in the confidence interval would be very high - well above 99%.

➡ If, in our sample, we had measured about **one quarter** of the population such as students in a school, our level of confidence that the true mean lies in the confidence interval would still be very high, but not as high as in the case above.

➡ If, in our sample, we had measured only a **few members** of the population such as the students in a school, our level of confidence that the true mean lies in the confidence interval **would not be very high**.

➡ For a fixed margin of error, the confidence level **increases** as the sample size increases.





A recent poll shows the Conservative candidate has 57% of the vote 19/20 times with a margin of error of 2.5%.

a) What is the confidence interval?

b) What is the confidence level?

c) What does this statement actually mean?





The heights of students are measured and the average is 64 inches with a margin of error of 3%. What is the confidence interval?

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## Questions

1. A telephone survey of 600 randomly selected people was conducted in an urban area. The survey determined that 76% of people, from 18 to 34 years of age, have a social networking account. The results are accurate within plus or minus 4 percent points, 19 times out of 20.
 

How can this result be interpreted, if the total population of 18 to 34 years olds is 92 500?
  
2. The results of the Vision Critical/Angus Reid poll also found that 60% of Canadians believe the images currently used for health warnings on cigarette packages are about right, while 24% wanted even more graphic imagery.
 

Calculate the range of people that would want more graphic images in a population of 30 000 adults. The margin of error for the survey was  $\pm 3.1\%$  and the confidence level was 95% (19/20).
  
3. Use confidence intervals to interpret the following statement and apply the result to a graduating class of 1400 students.
 

In a recent survey, 72% of post-secondary graduates indicated that they expected to earn at least \$6000/month by the time they were ready to retire. The survey is considered accurate within  $\pm 5.2\%$ , 19 times in 20.

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## Unit 5: Statistics Lesson 6

4. 200 randomly selected High School students were asked how much money they spent on computer related purchases over the past week. The sample mean for the 200 students was \$42.35.
- Did the selected students spend an average of \$42.35 on computer purchases last week?
  - What can be inferred by the result that the sample mean is \$42.35?
  - How could you be more confident in the sample mean?
5. Two different market research companies conducted a survey on the same issue. Company A used a 90% confidence level and company B used a 95% confidence level.
- If both companies used the same sample size, what does this imply about the margin of error for each survey?
  - If both companies used the same margin of error of  $\pm 3.5\%$ , what does this imply about the sample size for each survey?

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Example 1 :

VIOLET, THE STATS GIRL, STATES THAT 15% OF STUDENTS SMOKE IF THE MARGIN OF ERROR IS  $\pm 3\%$ .  
accurate 9 times out out 10.

a) Write her results as a C.I.

Solve: Low #  $\Rightarrow 15 - 3 = 12$   
upper #  $\Rightarrow 15 + 3 = 18$

$\therefore 12\%$  to  $18\%$

b) What is The Confidence Level.  
 $\frac{9}{10}$  times or 90%.

c) This means that there is a 90% likelihood that the actual mean lies between 12% and 18%.

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2. SAM THE STATS GUY  
SAYS THAT 73% of students  
dislike MATH. HER results  
are accurate to within  $\pm 5\%$ ,  
19 times out of 20.

a) Write as a C.I.

$$\text{Low} \Rightarrow 73 - 5 = 68$$

$$\text{High} \Rightarrow 73 + 5 = 78$$

$$\therefore 68\% - 78\%$$

b) Confidence Level

$$19 \text{ out of } 20 \text{ times} \Rightarrow \frac{19}{20} = 95\%$$

c) The actual mean lies between  
 $68\% - 78\%$ .

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#3 THE TELEGRAM reports that  
between 63% to 70% of teens  
OWN A pet. The results are  
accurate 9 times out of 10.

a) What is the margin of error?

$$\text{M.E.} \Rightarrow \frac{\text{High} - \text{Low}}{2} = \frac{70\% - 63\%}{2}$$

$$\therefore \text{ME} = \pm 3.5\% = \frac{7\%}{2} = 3.5\%$$

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