MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAMINATION	**Mathematics 1000**	WINTER 2015

COMPLETE THE FOLLOWING CAREFULLY AND CLEARLY:

(Please Print)

Surname: _____

Given Names: _____

MUN Number: _____

Circle the Name of Your Instructor Below:

J. Craighead   C-Z. Kuo   C. Leonard

Please note:

This exam has **TEN** pages, including this one.

The questions are to be answered in the spaces provided.

Under no circumstances may the candidate take this book from the examination room.

On no account are pages to be torn or removed from this book, unless specifically directed.

Candidates must not have in their possession books, notes or papers of any kind, unless specifically directed.

No electronic devices of any kind, including cell phones and MP3 players, are permitted at your desk. Calculators are **NOT** permitted.

	MARKS
12	1. _____
6	2. _____
7	3. _____
17	4. _____
6	5. _____
7	6. _____
6	7. _____
9	8. _____
17	9. _____
8	10. _____
5	11. _____
100	Total _____

FOR INSTRUCTOR'S USE ONLY

FINAL 55%	TERM 45%	TOTAL 100%	FINAL MARK	GRADE

Marks

1. Using methods learned in this course, evaluate the following limits. Show your work. Do not use l'Hôpital's rule

[3]   a) $\displaystyle \lim_{x \to -2} \frac{x^2 + 5x + 6}{x^2 - 2x - 8}$

[3]   b) $\displaystyle \lim_{x \to 3} \frac{4 - \sqrt{x^2 + 7}}{x^2 - 9}$

[3]   c) $\displaystyle \lim_{x \to 7^-} \frac{\left| x^2 - 49 \right|}{x - 7}$

[3] d) $\lim_{x \to 0} \dfrac{\sin^2(4x)}{3x^2 \cos^2(4x)}$

[6] 2. Let $f(x) = \begin{cases} \sqrt{4+x} & x < 0 \\ 1 & x = 0 \\ \dfrac{e^x + 1}{1-x} & x > 0 \end{cases}$

a) Use the Definition of Continuity to determine if $f(x)$ is continuous at $x = 0$. Classify any discontinuities as removable or non-removable.

b) Find any removable or non-removable discontinuities , if they exist.

[7] 3. Use the <u>DEFINITION OF DERIVATIVE</u> to find $f'(x)$ for $f(x) = \sqrt{2x-1}$.

4. Differentiate each function and make any appropriate simplifications.

[4] (a) $y = \tan^3\left(\sqrt{3x^3 - x^2}\right)$ DO NOT USE LOGARITHMIC DIFFERENTIATION.

[4]   (b) $y = \dfrac{2x^2 - 3x}{(4x - 3)^4}$ DO NOT USE LOGARITHMIC DIFFERENTIATION.

[4]   (c)   Use logarithmic to find y' for $y = \dfrac{e^{6x}}{\left(\sqrt{3x^2 - 2}\right)\left(3 + 5x^2\right)^4}$

[5]   (d) $y = (\cos x)^{(\sinh x)}$

[6] 5. Ship A leaves point P at the 10 AM and sail North at 15 kilometers an hour. Ship B leaves point P at 11 AM and sails west at 12 kilometers an hour. How fast is the distance between them changing at noon ?

[7] 6. A closed box is to be manufactured with its length 3 times its width and with a volume of 36 cubic meters. What dimensions will require the minimum material for construction?

[3]   7. a) Simplify the expression $\sin\left(\arctan\left(\frac{5}{6}\right)\right)$

[3]   b) If $f(x) = \arctan(3x^2)$ find $f'(x)$

[9]   8 a) Use implicit differentiation to find y' for $x^2 + 3xy^2 + y^3 = 9 + xy$

b) Find the equation of the tangent line at $(2,1)$

c) If $y = e^{\left(xy^2\right)}$ find y'

9. Given the following

$$f(x) = \frac{x-1}{(x-2)^2} \qquad f'(x) = \frac{-x}{(x-2)^3} \qquad f''(x) = \frac{2x+2}{(x-2)^4}$$

[3]   a) Find the vertical asymptotes of $f(x)$, if any. Justify your answer using Limits.

[3]   b) Find the horizontal asymptotes of $f(x)$, if any. Justify your answer using Limits

[2]   c) Find the x and y intercepts of the graph $f(x)$, if any.

[3]   d)Determine the intervals on which $f(x)$ is increasing or decreasing and classify any relative (local) extrema.

[3] e) Determine the intervals on which $f(x)$ is concave up or concave down and identify any inflection points.

[3] f) Sketch the graph of $f(x)$ on the axes provided. Label any inflection points and extrema.
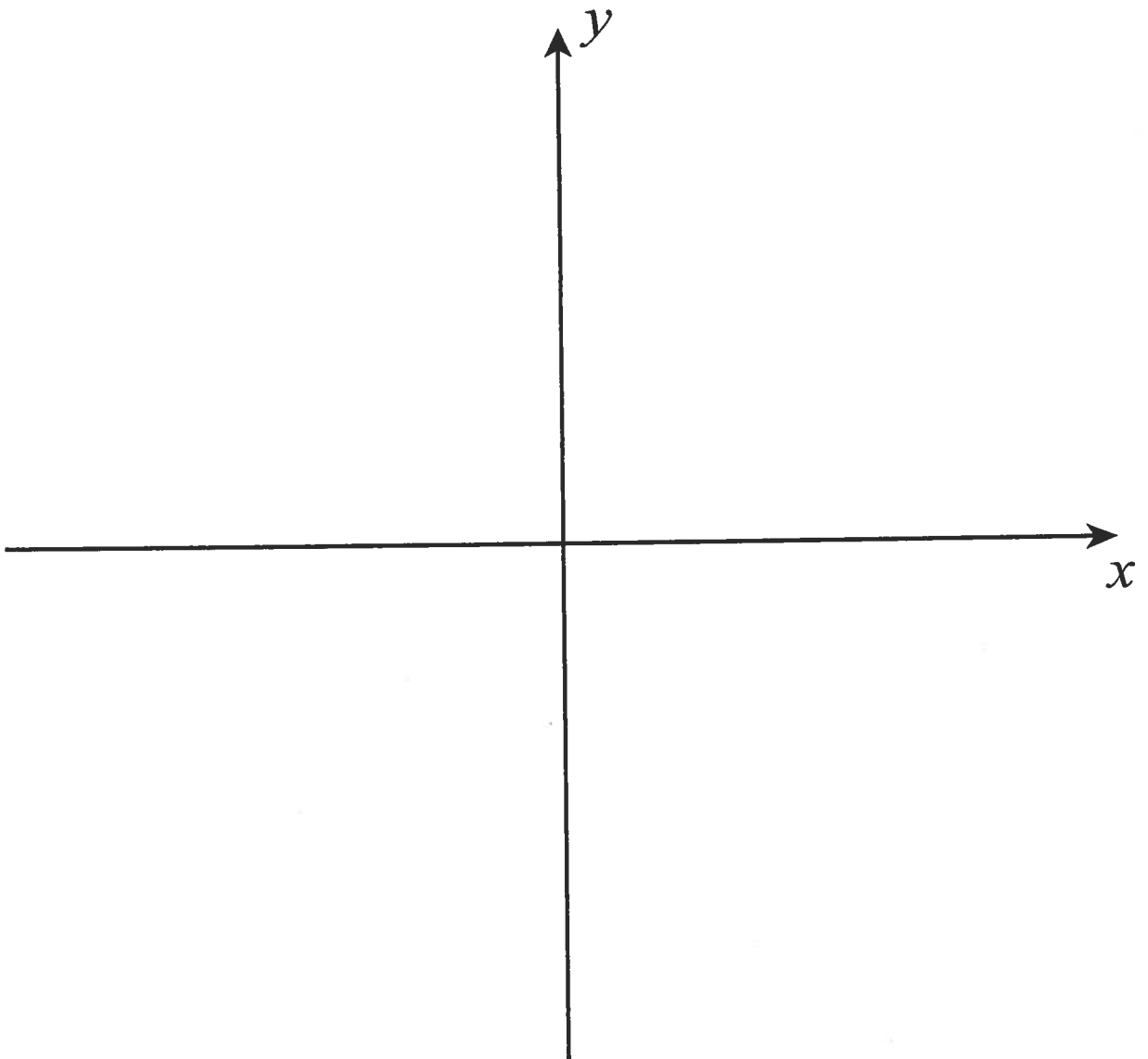
10. Use l'Hôpital's rule to find the following limits

[4]   a) $\lim\limits_{x\to\infty} x\sin\left(\frac{1}{x}\right)$

[4]   b) $\lim\limits_{x\to0^+} \left(1+4x\right)^{\frac{3}{x}}$

[5]   11.   Answer **ONE** of the following:

a)   Given $y = ax^3 + bx^2 + cx + d$, find a, b, c, d such that the y intercept = 3, critical numbers at $x = 3$ *and* $x = 2$ and an inflection point at $x = \dfrac{5}{2}$

or

b)   prove that if $y = \arcsin(x)$ Then $y' = \dfrac{1}{\sqrt{1-x^2}}$