MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAMINATION **Mathematics 1000** WINTER 2014

COMPLETE THE FOLLOWING CAREFULLY AND CLEARLY:

(Please Print)

Surname: _____

Given Names: _____

MUN Number: _____

Instructor: Austin Leonard Suvak Wang

Please note:

This exam has **EIGHT** pages of questions.

All calculators are strictly forbidden.

The questions are to be answered in the spaces provided.

Under no circumstances may the candidate take this book from the examination room.

On no account are pages to be torn or removed from this book, unless specifically directed.

Candidates must not have in their possession books, notes or papers of any kind, unless specifically directed.

No electronic devices of any kind, including cell phones and MP3 players, are permitted at your desk.

	MARKS
9	1. _____
4	2. _____
10	3. _____
5	4. _____
20	5. _____
10	6. _____
9	7. _____
5	8. _____
5	9. _____
8	10. _____
10	11. _____
5	12. _____
100	Total _____

FOR INSTRUCTOR'S USE ONLY

FINAL 55%	TERM 45%	TOTAL 100%	FINAL MARK	GRADE

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAMINATION | MATHEMATICS 1000 | Winter, 2014

Calculators are not permitted on this examination.

1. Evaluate each of the following limits, assigning ∞ or $-\infty$ where appropriate. You may not use L' Hospital's Rule.

[3] (a) $\lim_{x\to 2} \dfrac{3-\sqrt{x^2+5}}{x-2}$

[3] (b) $\lim_{x\to\infty} \dfrac{8x-5}{\sqrt[3]{8x^3+1}}$

[3] (c) $\lim_{x\to 1^+} \dfrac{x-3}{2x^2-5x+3}$

[4] 2. Find the horizontal and vertical asymptotes, if any, for the graph of the function

$$f(x) = \frac{(2x-1)^2}{4x^2-1}$$

[4] 3. (a) Determine whether the function

$$f(x) = \begin{cases} \dfrac{x^3 - 64}{x^2 - 16} & \text{if} \quad x \neq 4 \\ 0 & \text{if} \quad x = 4 \end{cases}$$

is continuous at $x = 4$. Justifly your answer using the definition of continuity.

[6] (b) Use the definition of continuity to find a and b, if possible, so that the function

$$f(x) = \begin{cases} ax + b & \text{if} \quad x < 2 \\ 6 & \text{if} \quad x = 2 \\ ax^2 - b & \text{if} \quad x > 2 \end{cases}$$

is continuous at $x = 2$.

[5] 4. Use the definition of the derivative to find the derivative of $f(x) = \dfrac{5}{3 - x^2}$.

5. Find and simplify the derivative of each of the following. Use logarithmic differentiation only where necessary:

[5] (a) $f(x) = \dfrac{\sin^2 2x}{1 + \cos^2 2x}$

[5] (b) $f(x) = \dfrac{e^{4x}}{\sqrt{1 - e^{4x}}}$

[5] (c) $f(x) = (1 + 2\ln x)^4 \ln^2 x$

[5] (d) $f(x) = (\sin x)^{\sin x}$

[5] 6. (a) Find and simplify the derivative of

$$f(x) = x\tan^{-1}\left(\frac{x}{4}\right) - 2\ln(x^2+16) \qquad [\text{ Note that } \tan^{-1}x = \arctan x\ ]$$

[5] (b) Find and simplify the second derivative y'' of the function given by

$$y = \sinh^2 3x$$

7. Find each of the following limits:

[4] (a) $\displaystyle\lim_{x\to 0}\frac{\cos x - \cos 3x}{x^2}$

[5] (b) $\displaystyle\lim_{x\to 0^+}(1+2\sin x)^{\frac{2}{x}}$

[5] 8. Find an equation of the normal line to the graph of the equation

$$2x - y \ln y = 4$$

at the point $(2, 1)$.

[5] 9. A helicopter leaves the ground at a point 30 metres horizontally away from an observer and rises vertically at a rate of 2 m/sec. At what rate is the distance between the observer and the helicopter changing 20 seconds after the helicopter leaves the ground?

[8] 10. A closed rectangular box is to have base with length three times its width. Find the dimensions of the box of least surface area if the volume is to be 288 cm^3.

[10] 11. Sketch the graph of $y = \dfrac{x^2}{(x+2)^2}$, giving intercepts, asymptotes, where increasing and where decreasing, any relative maximum and relative minimum points, where concave upward, where concave downward, and any inflection points. $\left[\text{Note: } y' = \dfrac{4x}{(x+2)^3}\right.$ and $\left. y'' = \dfrac{8(1-x)}{(x+2)^4}\right]$

[5] 12. Answer **one** of (a) or (b): Determine, with reasons, whether the given statement is True or False. No marks will be given for a correct answer without a valid justification.

100

(a) $\lim_{x\to\infty} \frac{x^n}{e^x} = 0$ for all positive integers n.

(b) The function $f(x) = |x - 2|$ is differentiable at $x = 2$.