

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAMINATION

Mathematics 1000

WINTER 2013

COMPLETE THE FOLLOWING CAREFULLY AND CLEARLY:

(Please Print)

Surname: _____

Given Names: _____

MUN Number: _____

Instructor: Craighead Leonard Suvak Wang

Please note:

This exam has **EIGHT** pages of questions.

All calculators are strictly forbidden.

The questions are to be answered in the spaces provided.

Under no circumstances may the candidate take this book from the examination room.

On no account are pages to be torn or removed from this book, unless specifically directed.

Candidates must not have in their possession books, notes or papers of any kind, unless specifically directed.

No electronic devices of any kind, including cell phones and MP3 players, are permitted at your desk.

MARKS	
14	1. _____
3	2. _____
7	3. _____
5	4. _____
18	5. _____
9	6. _____
10	7. _____
6	8. _____
5	9. _____
8	10. _____
10	11. _____
5	12. _____
100	Total _____

FOR INSTRUCTOR'S USE ONLY

FINAL 55%	TERM 45%	TOTAL 100%	FINAL MARK	GRADE

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MATHEMATICS 1000

Winter, 2013

Calculators are not permitted on this examination.

1. Evaluate each of the following limits, assigning ∞ or $-\infty$ where appropriate. You may not use L' Hospital's Rule.

[3] (a) $\lim_{x \rightarrow 9} \frac{3 - \sqrt{x}}{x^2 - 81}$

[3] (b) $\lim_{x \rightarrow -\infty} \frac{4x - 3}{\sqrt{4x^2 - 9}}$

(c) Given $f(x) = \frac{x^3 - 125}{2x^3 - 50x}$, find

[3] (i) $\lim_{x \rightarrow 5} f(x)$

[3] (ii) $\lim_{x \rightarrow -5^-} f(x)$

[2] (iii) $\lim_{x \rightarrow \infty} f(x)$

- [3] 2. Give the horizontal and vertical asymptotes, if any, for the graph of the function of #1(c).

[7] 3. Let

$$f(x) = \begin{cases} 3a \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ ae^x + 4 & \text{if } x > 0 \end{cases}$$

Find a so that $\lim_{x \rightarrow 0} f(x)$ exists. Is f continuous at $x = 0$ for this value of a ? If not, is the discontinuity at $x = 0$ removable. Verify your answers.

[5] 4. Use the definition of the derivative to find the derivative of $f(x) = \sqrt{ax + b}$, where a and b are constants.

5. Find and simplify the derivative of each of the following:

[3] (a) $f(x) = \frac{3}{1 + \sin^3 2x}$

[5] (b) $f(x) = \frac{1 + 2 \ln(\ln x)}{\ln^2 x}$

[5] (c) $f(x) = e^{4x} \sqrt{1 - e^{4x}}$

[5] (d) $f(x) = (\sec 4x)^{\frac{1}{x}}$

6. Find and simplify the derivative of each of the following:

[4] (a) $f(x) = \sin^{-1}(\sqrt{1 - e^{4x}})$ [Note that $\sin^{-1} x = \arcsin x$]

[5] (b) $f(x) = \frac{\sinh 2x}{1 + \cosh 2x}$

7. Find each of the following limits:

[5] (a) $\lim_{x \rightarrow 0} \frac{x \sin 2x}{1 - \cos 2x}$

[5] (b) $\lim_{x \rightarrow 0} (1 + \tan 4x)^{\frac{1}{2x}}$

- [6] 8. Find an equation of the normal line to the graph of the equation

$$2x^3 - 3x^2y = 2y^2 - 3.$$

at the point $(2, 3)$.

- [5] 9. A piece of ice in the shape of a right circular cylinder is melting. If its radius is decreasing at a rate of 2 cm/hr and its height is decreasing at a rate of 5 cm/hr, at what rate is the volume changing when the volume is 90π cm³ and the radius is 3 cm?

- [8] 10. A rectangular page is to contain 32 square inches of print, surrounded by 2 inch margins at the top and bottom of the page and 1 inch margins on each side. Find the dimensions of the page so that the least amount of paper is used.

- [10] 11. Sketch the graph of $y = \frac{9x}{(x+1)^2}$; giving intercepts, asymptotes, where increasing and where decreasing, any relative maximum and relative minimum points, where concave upward, where concave downward, and any inflection points. [Note: $y' = \frac{9(1-x)}{(x+1)^3}$ and $y'' = \frac{18(x-2)}{(x+1)^4}$]

[5] 12. Answer **one** of the following:

(a) If $f(x)$ is nonzero, differentiable and increasing on the open interval (a, b) , show that the function $g(x) = \frac{1}{f(x)}$ is decreasing on (a, b) .

(b) Given the equation $x^4 + y^4 = a^4$, where a is a constant, show that

$$y'' = -\frac{3a^4x^2}{y^7}$$