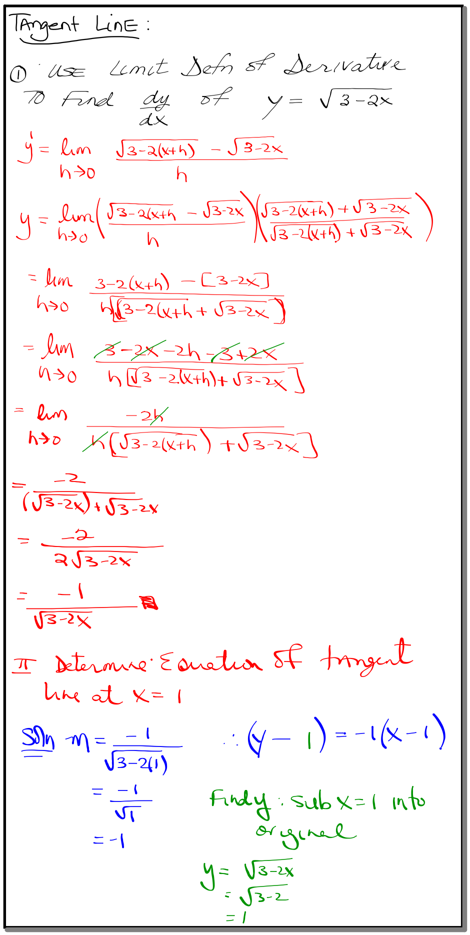

Tangent Line:

① Use Limit Defn of Derivative To Find $\frac{dy}{dx}$ of $y = \sqrt{3-2x}$

$$y' = \lim_{h\to 0} \frac{\sqrt{3-2(x+h)} - \sqrt{3-2x}}{h}$$

$$y = \lim_{h\to 0}\left(\frac{\sqrt{3-2(x+h} - \sqrt{3-2x}}{h}\right)\left(\frac{\sqrt{3-2(x+h)}+\sqrt{3-2x}}{\sqrt{3-2(x+h)}+\sqrt{3-2x}}\right)$$

$$= \lim_{h\to 0} \frac{3-2(x+h) - [3-2x]}{h[\sqrt{3-2(x+h} + \sqrt{3-2x}]}$$

$$= \lim_{h\to 0} \frac{\cancel{3} - \cancel{2x} - 2h - \cancel{3} + \cancel{2x}}{h[\sqrt{3-2(x+h)} + \sqrt{3-2x}]}$$

$$= \lim_{h\to 0} \frac{-2\cancel{h}}{\cancel{h}[(\sqrt{3-2(x+h}) + \sqrt{3-2x}]}$$

$$= \frac{-2}{(\sqrt{3-2x}) + \sqrt{3-2x}}$$

$$= \frac{-2}{2\sqrt{3-2x}}$$

$$= \frac{-1}{\sqrt{3-2x}}$$

II Determine Equation of tangent line at x = 1

Soln $m = \frac{-1}{\sqrt{3-2(1)}}$

$= \frac{-1}{\sqrt{1}}$

$= -1$

$\therefore (y - 1) = -1(x-1)$

Find y: sub x = 1 into original

$y = \sqrt{3-2x}$

$= \sqrt{3-2}$

$= 1$

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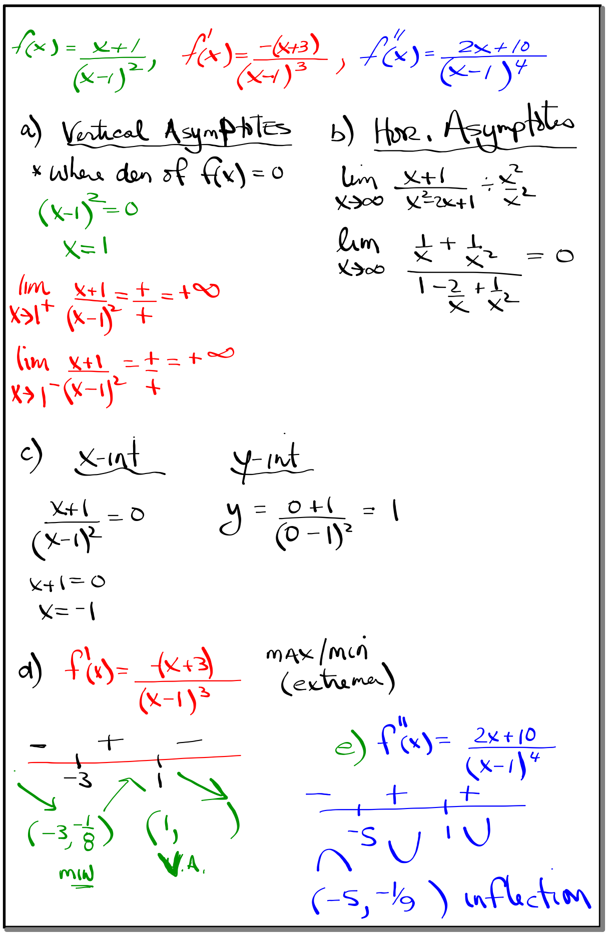

$f(x) = \frac{x+1}{(x-1)^2}$, $f'(x) = \frac{-(x+3)}{(x-1)^3}$, $f''(x) = \frac{2x+10}{(x-1)^4}$

a) Vertical Asymptotes

* where den of f(x) = 0

$(x-1)^2 = 0$

$x = 1$

$$\lim_{x\to 1^+} \frac{x+1}{(x-1)^2} = \frac{+}{+} = +\infty$$

$$\lim_{x\to 1^-} \frac{x+1}{(x-1)^2} = \frac{+}{+} = +\infty$$

b) Hor. Asymptotes

$$\lim_{x\to\infty} \frac{x+1}{x^2-2x+1} \div \frac{x^2}{x^2}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x} + \frac{1}{x^2}}{1 - \frac{2}{x} + \frac{1}{x^2}} = 0$$

c) x-int

$\frac{x+1}{(x-1)^2} = 0$

$x+1 = 0$

$x = -1$

y-int

$y = \frac{0+1}{(0-1)^2} = 1$

d) $f'(x) = \frac{-(x+3)}{(x-1)^3}$

max/min (extrema)

− + −

−3 1

$(-3, \frac{-1}{8})$ (1,)

min V.A.

e) $f''(x) = \frac{2x+10}{(x-1)^4}$

− + +

−5 1

$(-5, -\frac{1}{9})$ inflection

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$$\lim_{x\to 0} \frac{\sin 3x}{2x}$$

$$= \lim_{x\to 0} \boxed{\frac{\sin 3x}{3x}} \frac{(3\cancel{x})}{(2\cancel{x})}$$

$$= \frac{3}{2}$$

$$\lim_{x\to 0} \frac{\sin^2 4x}{x^2}$$

$$\lim_{x\to 0} \frac{(\sin 4x)(\sin 4x)(4\cancel{x})(4\cancel{x})}{(4x)(4x)(\cancel{x})(\cancel{x})}$$

$$= 16$$

③ $$\lim_{x\to 0} \frac{\sin^2(5x)}{x^2 \cos^2(2x)}$$

$$= \lim_{x\to 0} \frac{(\sin 5x)(\sin 5x)(5\cancel{x})(5\cancel{x})}{(5x)\ (5x)\ (\cancel{x})(\cancel{x})(\cos^2(2x))}$$

$$= \frac{25}{1}$$

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$$y = \ln\left[\frac{\sqrt{x}}{e^x \cdot \tan^3 x}\right]$$

$$y = \ln\sqrt{x} - \ln e^x - \ln \tan^3 x$$

$$y = \frac{1}{2}\ln x - x - 3\ln \tan x$$

$$y' = \frac{1}{2}\left(\frac{1}{x}\right) - 1 - 3\frac{1}{\tan x}\cdot \sec^2 x$$

$$y' = \frac{1}{2x} - 1 - 3\cot x \cdot \sec^2 x$$

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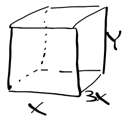
optimization



Top|Bot Sides Front|Back

$SA = 2(3x^2) + 2(3xy) + 2xy$

$A = 6x^2 + 6xy + 2xy$

$A = 6x^2 + 8xy$

$Vol = 36$

$36 = 3x(x)(y)$

$36 = 3x^2y$

$\frac{36}{3x^2} = y$

$\boxed{\frac{12}{x^2} = y}$

$\therefore A = 6x^2 + 8x\left(\frac{12}{x^2}\right)$

$A = 6x^2 + \frac{96}{x}$

$A' = 12x - \frac{96}{x^2}$

$A' = 0$ $12x - \frac{96}{x^2} = 0$

$12x = \frac{96}{x^2}$

$\frac{12x^3}{12} = \frac{96}{12}$

$x^3 = 8$

$x = \sqrt[3]{8} = 2$

$A = 6(2)^2 + \frac{96}{(8)}$

$A = 24 + 48$

$A = 72$

Two Choices

$A'' = 12 + \frac{192}{x^3}$

$A''(2) = 12 + \frac{192}{(2)^3} = 12 + 24 = 36 \Rightarrow$ Pos U

OR

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$y = \overset{f}{5^x} \cdot \overset{g}{\tan^{-1}(2x)}$

$y' = \overset{f'}{(5^x \cdot \ln 5)} \overset{g}{\tan^{-1}(2x)} + \overset{g'}{\frac{1}{1+(2x)^2}(2)} \cdot \overset{f}{5^x}$

$y' = 5^x \cdot \ln 5 \tan^{-1}(2x) + \frac{2 \cdot 5^x}{1+4x^2}$

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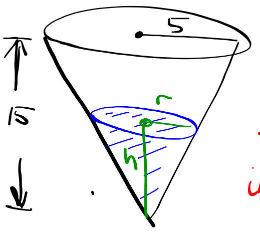
Related Rates



Water leaks out at a rate of 20 cm^3/min.

How is the height of the water changing at the instant the height is 6 cm?

$\frac{dv}{dt} = -20\,cm^3/min$

$\left.\frac{dh}{dt}\right|_{h=6}$

$\frac{15}{h} = \frac{5}{r}$

$\frac{5h}{15} = \frac{15r}{15}$

$\boxed{\frac{h}{3} = r}$

$V = \frac{1}{3}\pi r^2 h$

$\therefore V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

$V = \frac{\pi}{27}h^3$

$\frac{dv}{dt} = \frac{\pi}{9}h^2\cdot\frac{dh}{dt}$

$-20 = \frac{\pi}{9}(6)^2\frac{dh}{dt}$

$\frac{-20}{4\pi} = \frac{4\pi}{4\pi}\frac{dh}{dt}$

$\boxed{\frac{-5}{\pi} = \frac{dh}{dt}}$

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$\pi(x^2-y^2) = \cos(\pi y)$ at $\left(\frac{3}{2},\frac{3}{2}\right)$

$\pi x^2 - \pi y^2 = \cos(\pi y)$

$2\pi x - 2\pi y\frac{dy}{dx} = -\sin(\pi y)\cdot\pi\cdot\frac{dy}{dx}$

$2\cancel{\pi}x = 2\cancel{\pi}y\frac{dy}{dx} - \sin(\pi y)\cancel{\pi}\frac{dy}{dx}$

$2x = \frac{dy}{dx}\left[2y - \sin(\pi y)\right]$

$\frac{2x}{2y-\sin(\pi y)} = \frac{dy}{dx}$

$\therefore$ at $\left(\frac{3}{2},\frac{3}{2}\right) \Rightarrow \frac{dy}{dx} = \frac{2(3/2)}{2(3/2) - \sin\frac{3\pi}{2}}$

$= \frac{3}{3 - -1}$

$= \frac{3}{4}$

$\therefore \left(y-\frac{3}{2}\right) = \frac{3}{4}\left(x-\frac{3}{2}\right)$

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