Composition of Functions

1. Given $f(x) = 3x - 5$ and $g(x) = -2x + 7$, find each value.

a) $f(g(1))$	**b)** $f(g(-4))$	**c)** $g(f(-2))$	**d)** $g(f(3))$
$= f(5)$	$= f(15)$	$= g(-11)$	$g(4)$
$= 10$	$= 40$	$= 29$	$= -1$

2. Given $f(x) = 8 - 2x$ and $g(x) = x^2 + 3x + 4$, find each value.

a) $f(g(1))$	**b)** $f(g(-4))$	**c)** $g(f(-2))$	**d)** $g(f(3))$
$f(8) = -8$	$f(8) = -8$	$g(12) = 184$	$g(2) = 14$

3. Given $f(-1) = 7, f(7) = 5, f(3) = 0, g(-1) = 3, g(7) = -1,$ and $g(5) = -2$, find each value.

a) $f(g(7))$	**b)** $f(g(-1))$	**c)** $g(f(-1))$	**d)** $g(f(7)) = g(5) = -2$
$f(-1) = 7$	$f(3) = 0$	$g(7) = -1$	

4. Use the graph to find each value.

$g(4) = 4$ $g(5) = 6$

a) $g(f(-1))$ **b)** $g(f(-2))$

$f(-2) = 5$

c) $f(g(1))$ **d)** $f(g(2))$ $f(0) = 3$

5. If $f(x) = 2x - 9$ and $g(x) = x^2 + 6$,

determine each of the following.

a) $f(g(a))$
$f(a^2 + 6) = 2(a^2 + 6) - 9$
$= 2a^2 + 3$

b) $g(f(a))$
$(2a - 9)^2 + 6$
$4a^2 - 36a + 87$

c) $f(g(x))$
$= 2x^2 + 3$

d) $g(f(x))$
$4x^2 - 36x + 87$

e) $f(f(x))$
$2(2x - 9) - 9 \to 4x - 27$

f) $g(g(x))$
$(x^2 + 6)^2 + 6 = x^4 + 12x^2 + 42$

6. Consider $f(x) = x^2 - 5$ and $g(x) = \sqrt{x + 2}$.

a) Determine $y = f(g(x))$. $f(\sqrt{x+2}) = (\sqrt{x+2})^2 - 5 = x + 2 - 5 = x - 3$ ■

b) Sketch the graph of $y = f(g(x))$.

**c)** State the domain and range of $y = f(g(x))$.
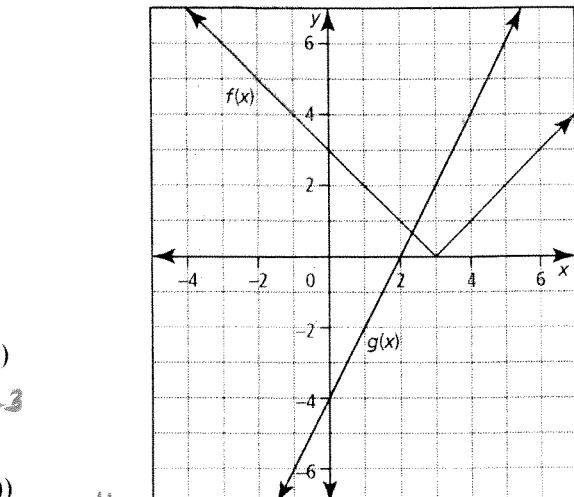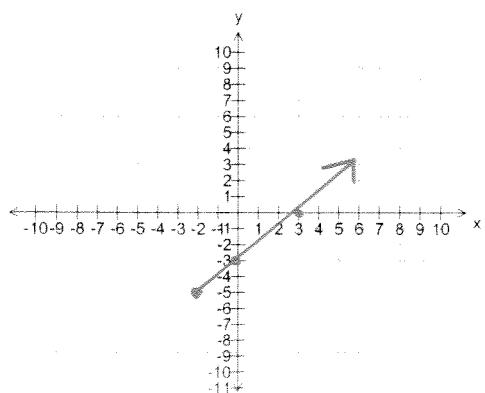
D: $x \geq -2$ or $[-2, \infty)$

R: $y \geq -5$ or $[-5, \infty)$

7. Given $f(x) = \sqrt{x+2}$ and $g(x) = x + 5$, sketch the graph of each composite function. Then, determine the domain and range of each composite function.

$g(f(x))$ $g(\sqrt{x+2}) = (\sqrt{x+2}) + 5$

a) $f(g(x))$

$(-2, 5)$

$f(g(x)) = \sqrt{(x+5)+2}$
$= \sqrt{x+7}$
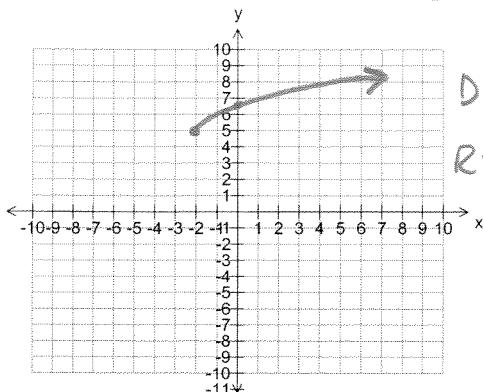
b) ~~$g(f(x))$~~

D: $\{x \mid x \geq 2\}$

R: $\{y \mid y \geq 5\}$
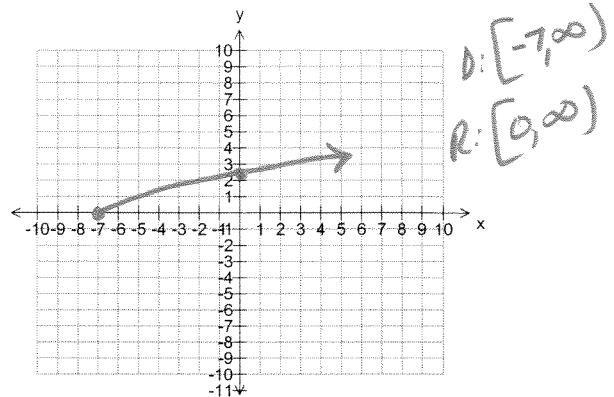
D: $[-7, \infty)$

R: $[0, \infty)$

8. For each pair of functions, $f(x)$ and $g(x)$, determine $f(g(x))$ and $g(f(x))$.

a) $f(x) = 3x - 1$ $g(x) = x^2 + 2$

$f(x^2+2) = 3(x^2+2) - 1$
$= 3x^2 + 6 - 1$
$= 3x^2 + 5$

$g(3x-1) = (3x-1)^2 + 2$
$= 9x^2 - 6x + 1 + 2$
$= 9x^2 - 6x + 3$

b) $f(x) = x^2 - 4$ $g(x) = 5x + 7$

$f(5x+7) = (5x+7)^2 - 4$
$= 25x^2 + 70x + 45$

$g(x^2-4) = 5(x^2-4) + 7$
$= 5x^2 - 13$

c) $f(x) = x^2 - x$ $g(x) = x^2 + x$

$f(x^2+x) = (x^2+x)^2 - x$
$= x^4 + 2x^3 + x^2 - x$

$g(x^2-x) = (x^2-x)^2 + x$
$= x^4 - 2x^3 + x^2 + x$

d) $f(x) = x^2 - 9$ $g(x) = \sqrt{x+4}$

$f(\sqrt{x+4}) = (\sqrt{x+4})^2 - 9$
$= x + 4 - 9$
$= x - 5$

$g(x^2-9) = \sqrt{(x^2-9)+4}$
$= \sqrt{x^2-5}$

9. If $h(x) = (f \circ g)(x)$, determine $g(x)$.

a) $h(x) = x - 4$ and $f(x) = x^2$

$[g(x)]^2 = x - 4$

$g(x) = \pm\sqrt{x-4}$

b) $h(x) = x^2 + 6x + 5$ and $f(x) = x^2 - 4$

$[g(x)]^2 - 4 = x^2 + 6x + 5$

$[g(x)]^2 = x^2 + 6x + 9$

$g(x) = \pm\sqrt{(x+3)^2}$

$g(x) = \pm(x+3)$