BRITISH COLUMBIA

# Pre-Calculus 12
## Resource Exam B
Exam Booklet II

Multiple-Choice and Written-Response Questions
**Calculator NOT Permitted**

**Contents: 23 pages**
30 multiple-choice and 2 written-response questions in Exam Booklet II

**Examination: 2 hours**
**Additional Time Permitted: 60 minutes**

**MULTIPLE-CHOICE QUESTIONS**
**(Calculator NOT permitted)**

**Value: 53 marks**

**INSTRUCTIONS: No calculator may be used for this section of the examination.**
For each question select the **best** answer.

15. Determine the equivalent expression for ${}_{22}C_8$.

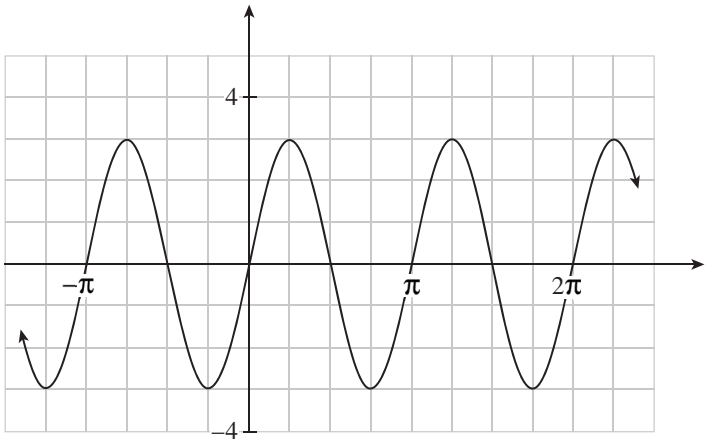
A. ${}_8C_{22}$

B. ${}_{20}C_6$

C. ${}_{22}C_{14}$

D. $\frac{{}_{22}P_8}{14}$

16. The graph of $y = a \sin bx$ is shown below. Determine the values of $a$ and $b$.




A. $a = -3;\ b = 2$

B. $a = -3;\ b = \pi$

C. $a = 3;\ b = 2$

D. $a = 3;\ b = \pi$

17. The terminal arm of angle $\theta$ in standard position intersects the unit circle at the point $(x,\ y)$. Which expression represents $\cot\theta$?

A. $\frac{y}{x}$

B. $\frac{x}{y}$

C. $x$

D. $y$

18. Which expression represents the measure of all angles in radians that are coterminal with angle $\theta$?

A. $2\pi + n\theta$, $n \in I$

B. $\theta + \frac{\pi}{2}n$, $n \in I$

C. $\theta + \pi n$, $n \in I$

D. $\theta + 2\pi n$, $n \in I$

It is acceptable to state the domain as $n \in I$, or $n \in Z$, or $n$ is an integer.

19. Determine the range of the function $y = -5\sin 2x - 3$.

A. $-8 \le y \le 2$

B. $-8 \le y \le -2$

C. $-5 \le y \le 5$

D. $-2 \le y \le 8$

20. Determine the general solution for $\sin 4x = -1$.

A. $x = \frac{\pi}{8} + \frac{\pi n}{2}$ where $n$ is an integer

B. $x = \frac{3\pi}{8} + \frac{\pi n}{2}$ where $n$ is an integer

C. $x = \frac{\pi}{8} + 2\pi n$ where $n$ is an integer

D. $x = \frac{3\pi}{8} + 2\pi n$ where $n$ is an integer

21. Chantal simplified the expression $\frac{\csc\theta + \sec\theta}{\sin\theta + \cos\theta}$ as shown below. In which step is Chantal's first error?

| Steps | |
|---|---|
| 1. | $\frac{\frac{1}{\sin\theta} + \frac{1}{\cos\theta}}{\sin\theta + \cos\theta}$ |
| 2. | $\frac{\frac{\cos\theta + \sin\theta}{\sin\theta}}{\sin\theta + \cos\theta}$ |
| 3. | $\left(\frac{\cos\theta + \sin\theta}{\sin\theta}\right)\left(\frac{1}{\sin\theta + \cos\theta}\right)$ |
| 4. | $\frac{1}{\sin\theta}$ |

A. 1
B. 2
C. 3
D. 4

22. Determine all non-permissible values for the expression $\frac{\sec x}{2\sin x + 1}$, in the interval $0 \le x < 2\pi$

A. $x = \frac{\pi}{2}, \frac{3\pi}{2}$

B. $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

C. $x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

D. $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Students should recognize that in trigonometry, stating non-permissible values is different than stating restrictions.

23. The function $h = -5\cos\frac{\pi}{60}t + 6$ gives Cassandra's height, $h$ metres, above the ground when she is riding a Ferris wheel after $t$ seconds. Determine the length of time for one rotation and the lowest point on the Ferris wheel.

A. 60 sec, 6 m
B. 60 sec, 1 m
C. 120 sec, 6 m
D. 120 sec, 1 m

24. Determine the number of solutions for $\sin^2 x(\csc x + 1) = 0$ in the interval $0 \le x < 2\pi$ with the correct reasoning.

A. There are three solutions because $\sin^2 x = 0$ has two solutions and $\csc x + 1 = 0$ has one solution in the interval $0 \le x < 2\pi$.

B. There are three solutions because $\sin^2 x = 0$ has one solution and $\csc x + 1 = 0$ has two solutions in the interval $0 \le x < 2\pi$.

C. There is one solution because $\sin^2 x = 0$ has to be rejected and $\csc x + 1 = 0$ has one solution in the interval $0 \le x < 2\pi$.

D. There is one solution because $\sin^2 x = 0$ has one solution and $\csc x + 1 = 0$ has to be rejected in the interval $0 \le x < 2\pi$.

25. Two students, Yuri and Rubin, solved the exponential equation $2^{x+1} = 3$ as shown below.

| **Yuri's Solution** | **Rubin's Solution** |
|---|---|
| $2^{x+1} = 3$<br>$\log 2^{x+1} = \log 3$<br>$(x+1)\log 2 = \log 3$<br>$x\log 2 + \log 2 = \log 3$<br>$x\log 2 = \log 3 - \log 2$<br>$x = \dfrac{\log 3 - \log 2}{\log 2}$ | $2^{x+1} = 3$<br>$x + 1 = \log_2 3$<br>$x = \log_2 3 - 1$ |

Which statement is true?

A. Yuri is incorrect, Rubin is incorrect.
B. Yuri is incorrect, Rubin is correct.
C. Yuri is correct, Rubin is incorrect.
D. Yuri is correct, Rubin is correct.

26. What is the best estimation of $\log_3 30$ ?

A. 3.1
B. 3.4
C. 3.6
D. 3.9

It is expected that students will use benchmarks to estimate the value of a logarithm. For example, students should know that $\log_3 27 = 3$ and $\log_3 81 = 4$, therefore $\log_3 30$ should be close to 3.

27. Solve for $x$: $\log_2 3 = 2\log_8 x$

A. $3^{\frac{2}{3}}$
B. $3^{\frac{3}{2}}$
C. $2^{\frac{2}{3}}$
D. $2^{\frac{3}{2}}$

28. Determine the Richter scale reading for an earthquake that is 5 times more intense than another earthquake that measures 4.0 on the Richter scale.

A. 9
B. 20
C. $4 + \log 5$
D. $5 + \log 4$

29. Determine the domain of the function $y = \log\left(4 - x^2\right)$.

A. $-2 < x < 2$
B. $-2 \le x \le 2$
C. $x < -2,\ x > 2$
D. $x \le -2,\ x \ge 2$

It is expected that students are able to solve a quadratic inequality.

30. Explain how the graph of $y - 5 = f(x)$ is related to the graph of $y = f(x)$.

A. It is the graph of $y = f(x)$ translated 5 units up.
B. It is the graph of $y = f(x)$ translated 5 units down.
C. It is the graph of $y = f(x)$ translated 5 units to the left.
D. It is the graph of $y = f(x)$ translated 5 units to the right.

31. For which of the following functions is $f(x) = f(-x)$?

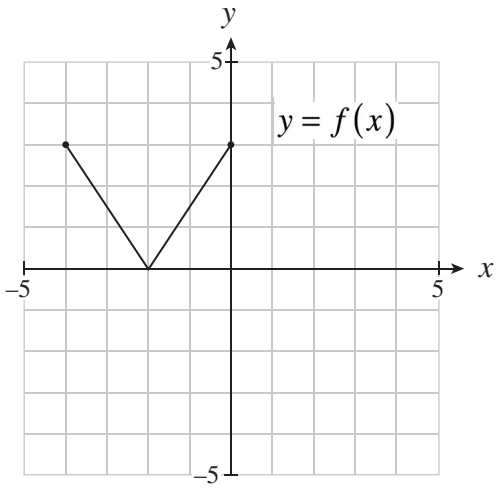
| | |
|---|---|
| I. | $y = \sin x$ |
| II. | $y = \cos x$ |
| III. | $y = (x-3)^2$ |
| IV. | $y = x^2 + 3$ |

A. I, III only
B. I, IV only
C. II, III only
D. II, IV only

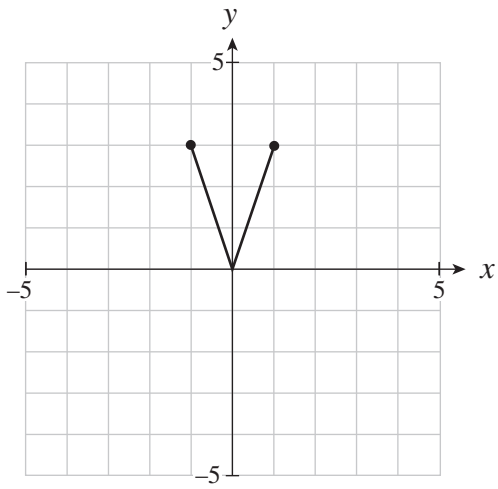
32. The point $\mathrm{P}(-3, -8)$ is on the graph of $y = f(x)$. Which point must be on the graph of $y = -f(x-5)$?

A. $(-8, -8)$
B. $(-8, 8)$
C. $(2, 8)$
D. $(8, -8)$

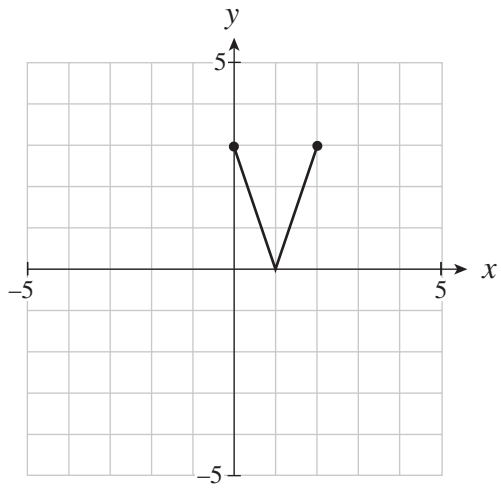
33. The graph of $y = f(x)$ is shown below. Which graph represents $y = f(2x - 4)$.



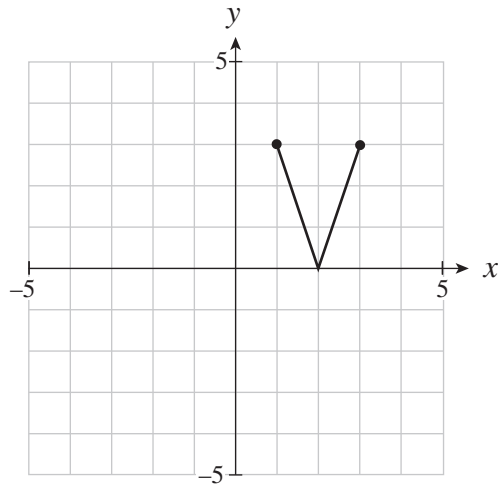

A.



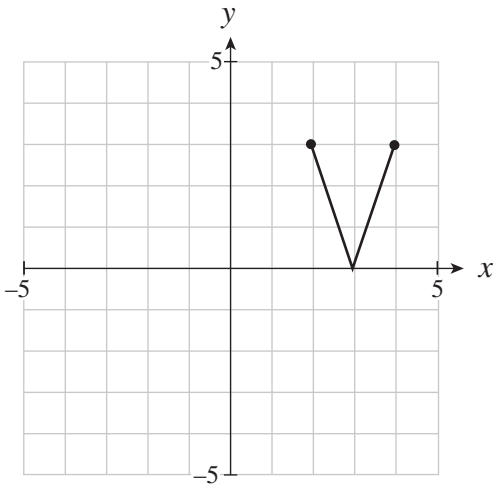

B.




C.




D.



34. A cubic polynomial function $f$ has zeros $\{-3,\ 0,\ 2\}$. Which restriction on the domain of $f$ will allow its inverse to be a function?

A. $x > -3$
B. $x > 0$
C. $x < 0$
D. $x > 2$

35. Raj used synthetic division to divide a polynomial $f(x)$ by $x - 2$ as shown below.

$$\begin{array}{r|llll} 2 & 1 & -3 & k & -5 \\ \hline & & & & -1 \end{array}$$

Determine the value of $k$ that will give a remainder of $-1$ as shown in the table.

A. 1
B. 4
C. 5
D. 6

In this question, because we are dividing by $x - 2$, the number 2 is used outside the synthetic division box so that the operation of addition can be used to find the coefficients of the quotient and remainder.

36. Compare the graphs of the two functions at $x = 2$.

$$f(x) = x(x-2)^3(x+2) \text{ and } g(x) = x(x-2)^2(x+2)$$

A. The graph of $f(x)$ crosses the $x$-axis at $x = 2$ and the graph of $g(x)$ just touches the $x$-axis at $x = 2$ but does not cross it.

B. The graph of $f(x)$ just touches the $x$-axis at $x = 2$ but does not cross it and the graph of $g(x)$ crosses the $x$-axis at $x = 2$.

C. The graph of $f(x)$ crosses the $x$-axis at $x = 2$ and the graph of $g(x)$ crosses the $x$-axis at $x = 2$.

D. The graph of $f(x)$ just touches the $x$-axis at $x = 2$ but does not cross it and the graph of $g(x)$ just touches the $x$-axis at $x = 2$ but does not cross it.

It is important that students understand and can explain how the multiplicity of a zero of a polynomial function affects the graph.

37. Given the functions $f(x) = x + 3$ and $g(x) = x^2 - 4$, determine the value of $(f + g)(-2)$.

A. 0
B. 1
C. 3
D. 5

☞ Students should be familiar with different forms for operations on functions.

$$\frac{f(x)}{g(x)} = \left(\frac{f}{g}\right)(x)$$

$$f(x) + g(x) = (f + g)(x)$$

$$f(x)g(x) = (fg)(x)$$

$$f(x) - g(x) = (f - g)(x)$$

38. For which of the following functions is $f(f(x)) = x$, for all values of $x$ in the domain of $f$?

| | |
|---|---|
| I. | $f(x) = x$ |
| II. | $f(x) = -x$ |
| III. | $f(x) = \frac{1}{x}$ |

A. I and II only
B. I and III only
C. II and III only
D. I, II and III

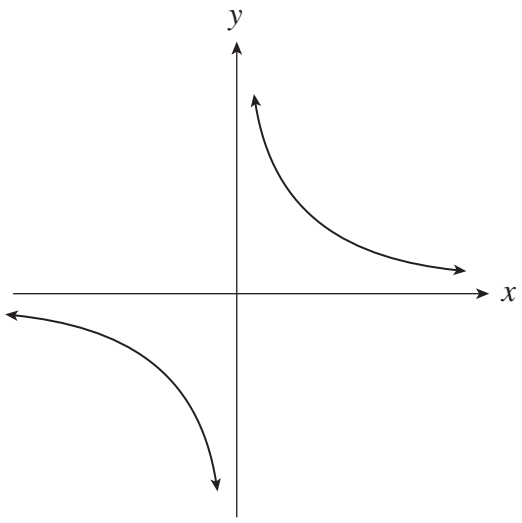
☞ The intent of this question is for students to realize that $f(f(x)) = x$ occurs when $f(x) = f^{-1}(x)$. When a function $f(x)$ is the inverse of itself, the graph of $y = f(x)$ is symmetric to the line $y = x$.

39. A polynomial function $f$ has zeros at $1,\ -1,$ and 2. Given the function $g(x)=\dfrac{x+1}{x-2}$, determine the domain of the function $h(x)=\dfrac{f(x)}{g(x)}$.

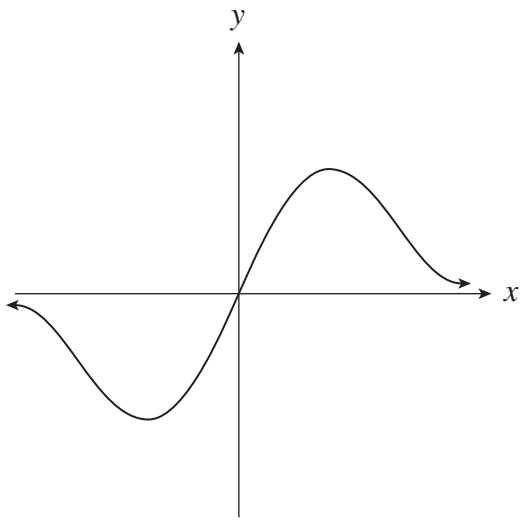
A. all real numbers
B. all real numbers, $x \neq 2$
C. all real numbers, $x \neq -1$
D. all real numbers $x \neq -1$ and $x \neq 2$

40. Given $f(x)=x$ and $g(x)=\dfrac{1}{x}$, which graph best represents $y=f(x)+g(x)$?

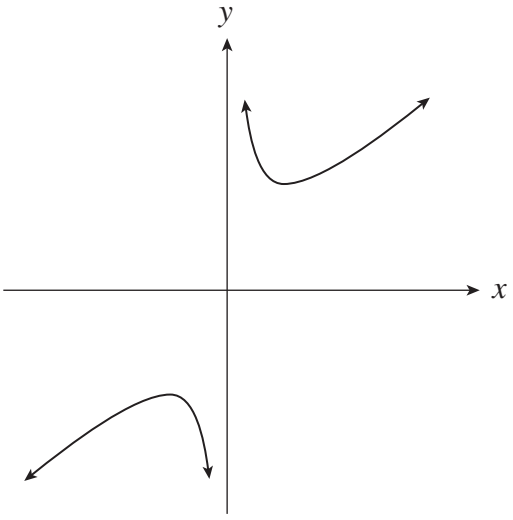
A.



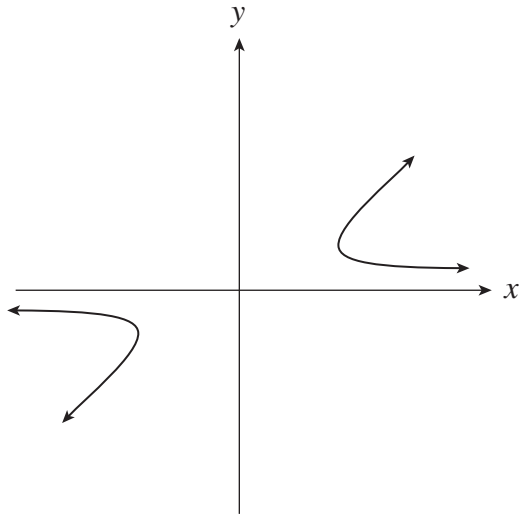

B.



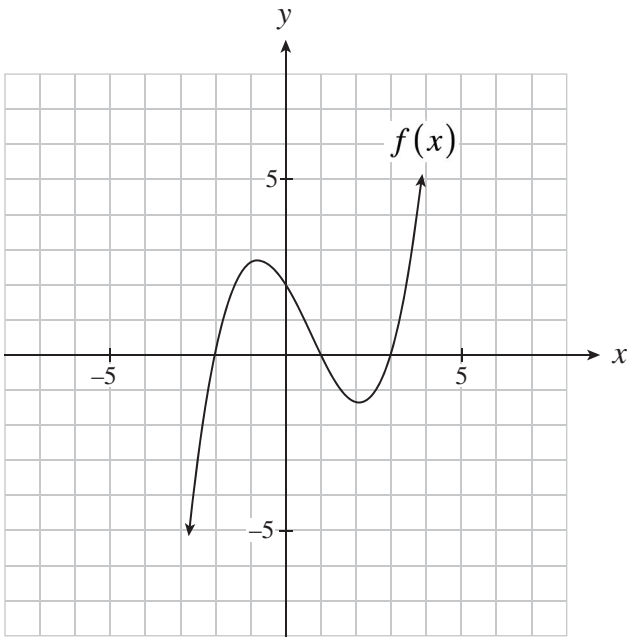

C.

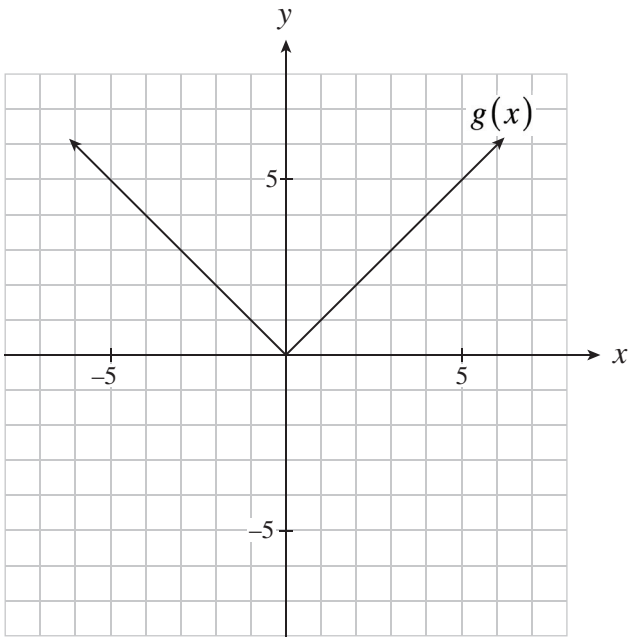



D.



41. The graphs of $f(x)$ and $g(x)$ are given below. Determine $f(g(-3))$.




A. $-6$
B. $-2$
C. $0$
D. $3$

Students should be familiar with the notation $f(g(x))$ is equivalent to $(f \circ g)(x)$.

42. Determine the equations of all asymptotes for the graph of $y + 2 = \frac{1}{x-1}$.

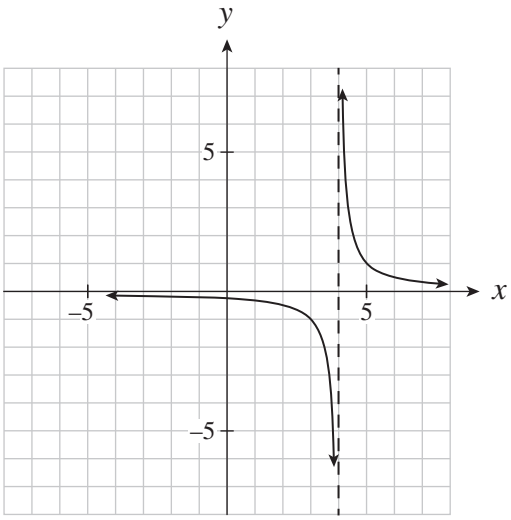
A. $x = -1,\ y = -2$
B. $x = -1,\ y = 2$
C. $x = 1,\ y = -2$
D. $x = 1,\ y = 2$

This could be done in a variety of ways. For example:

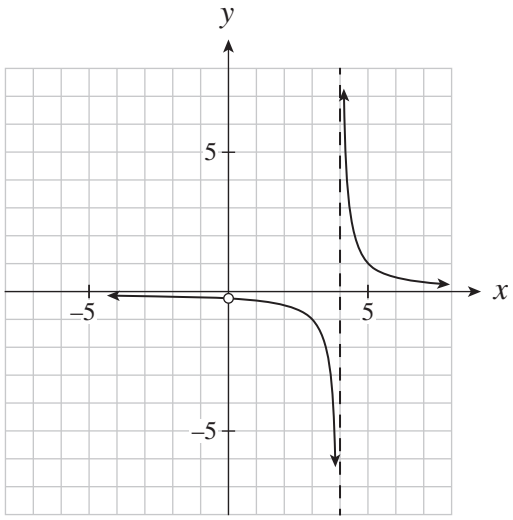
- transformations on the function $y = \frac{1}{x}$
- rewrite the function as a rational function $y = \frac{-2x-3}{x-1}$, and consider the end behaviour model to determine the horizontal asymptote.

43. Which of the following best represents the graph of the rational function $y = \dfrac{x}{x^2 - 4x}$?

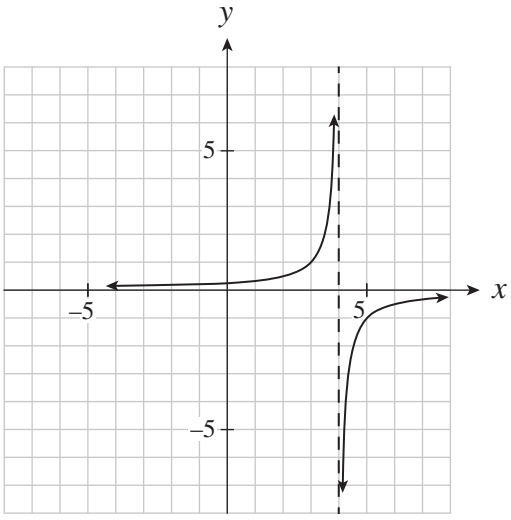
A.



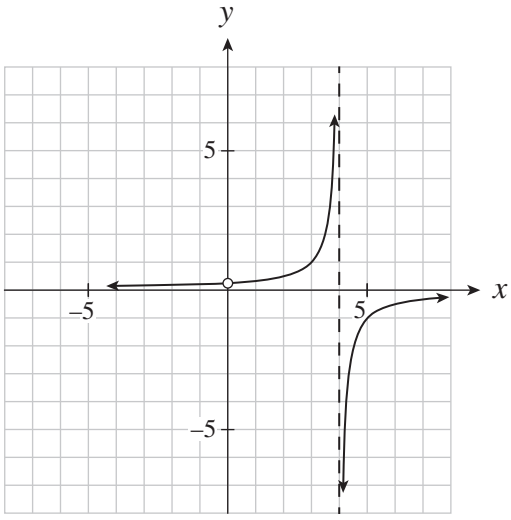

B.




C.




D.



44. For the function $f(x) = \dfrac{x^2 - 4}{x^2 - 2x}$, which of the following statements explain the behaviour of the graph of $f$ for the values of a variable near a non-permissible value?

A. When $x$ is close to 2 on either side, $f$ is close to 2.
When $x$ is just to the right of 0, $f$ is a large positive value.
When $x$ is just to the left of 0, $f$ is a large negative value.

B. When $x$ is close to 2 on either side, $f$ is close to 4.
When $x$ is just to the right of 0, $f$ is a large positive value.
When $x$ is just to the left of 0, $f$ is a large negative value.

C. When $x$ is close to 2 on either side, $f$ is close to 2.
When $x$ is just to the right of 0, $f$ is a large negative value.
When $x$ is just to the left of 0, $f$ is a large positive value.

D. When $x$ is close to 2 on either side, $f$ is close to 4.
When $x$ is just to the right of 0, $f$ is a large negative value.
When $x$ is just to the left of 0, $f$ is a large positive value.

Students are expected to be able to find the location of the hole, also called the point of discontinuity.

**This is the end of the Multiple-Choice section.**
**Answer the remaining Written-Response questions directly in this booklet.**

## WRITTEN-RESPONSE QUESTIONS
## (Calculator NOT permitted)

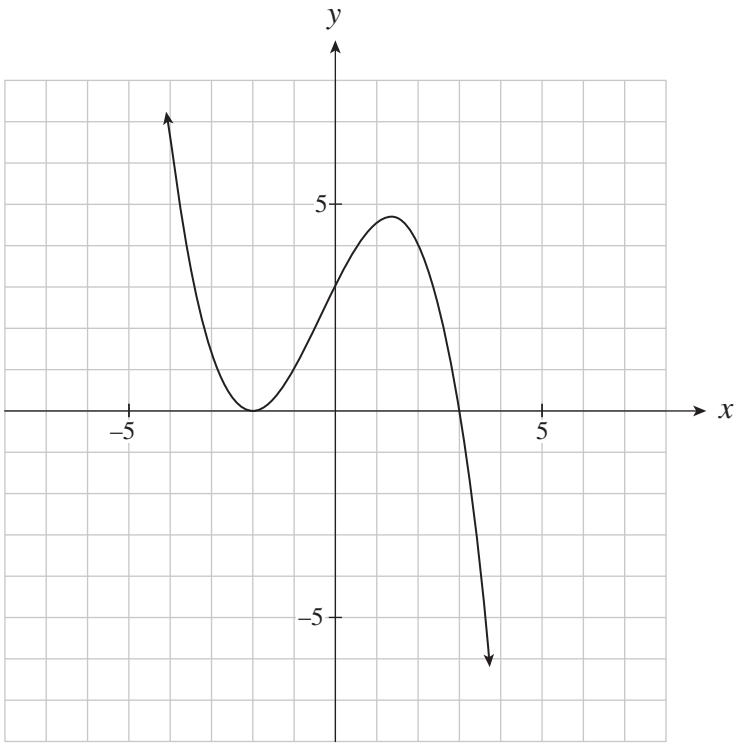
**Value: 8 marks**

**INSTRUCTIONS:** Answer the following questions in the space provided.

Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question.

**Full marks will NOT be given for a final answer only.**

1. Determine an equation for the cubic polynomial function graphed below.
   Leave answer in factored form. **(4 marks)**



2. Explain the relationship between the exponential function $f(x) = 2^x + 1$ and its inverse.

   Provide an answer that includes an algebraic analysis and describes graphical characteristics. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language. **(4 marks)**

**End of Examination Booklet II**