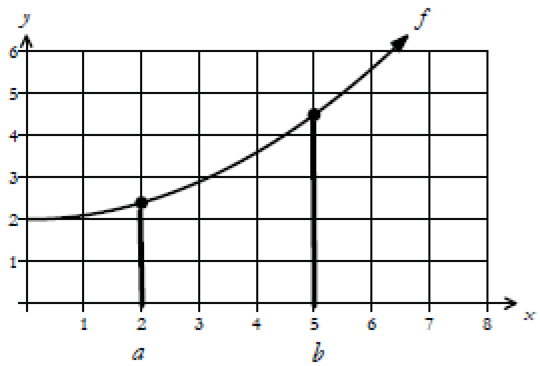
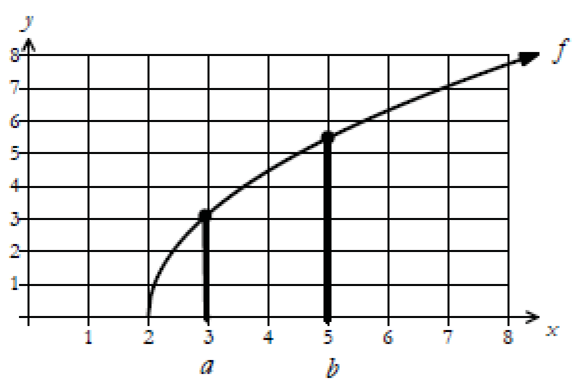
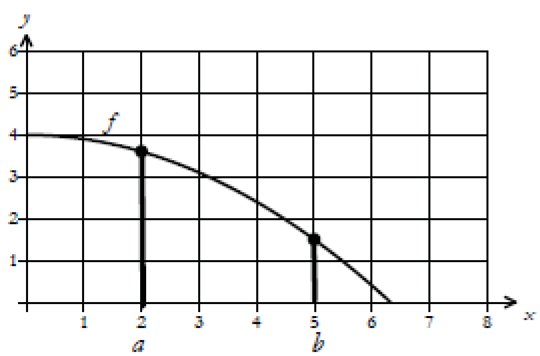
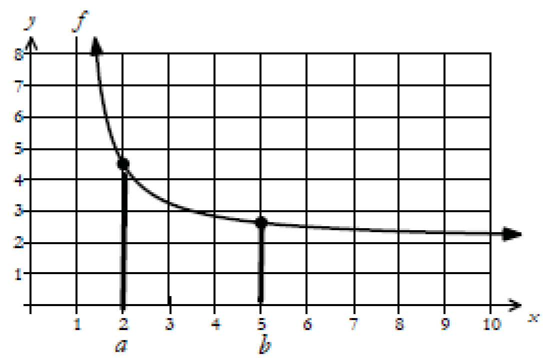
C6.1 *Use $f'(x)$ to identify the critical numbers, relative and absolute extrema and intervals of increase and decrease.*





Moving left to right along the graph, a function, f, is increasing if the y-coordinate increases in value (ie. if $f(b) > f(a)$ whenever $b > a$).

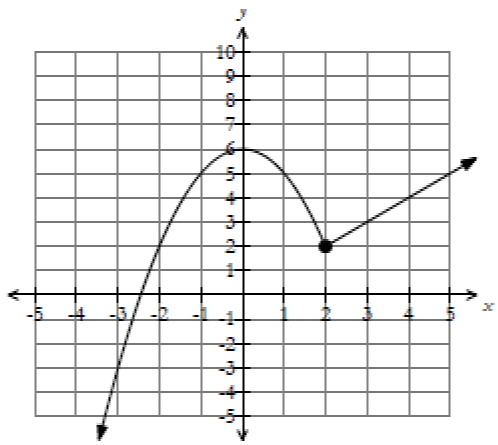




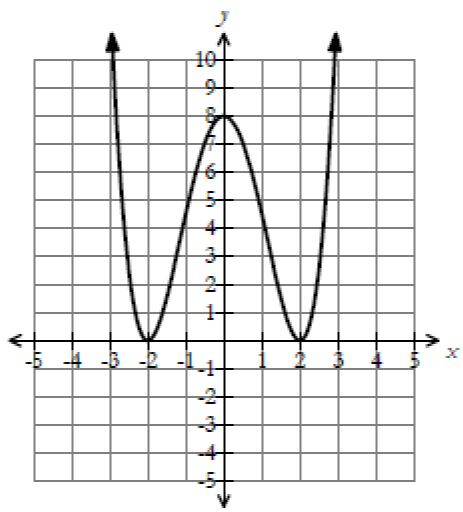
Moving left to right along the graph, a function, f, is decreasing if the y-coordinate decreases in value (ie. if $f(b) < f(a)$ whenever $b > a$).

What are the intervasl of increase/decrease?

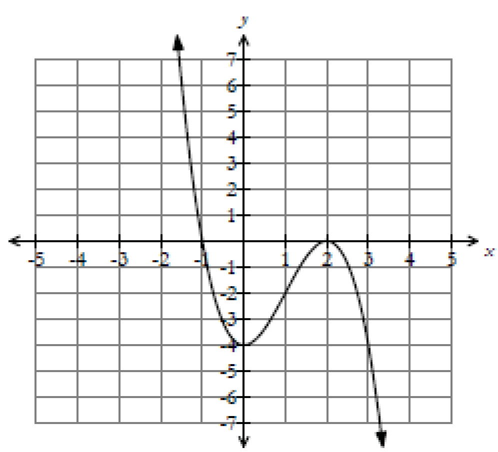
(i)

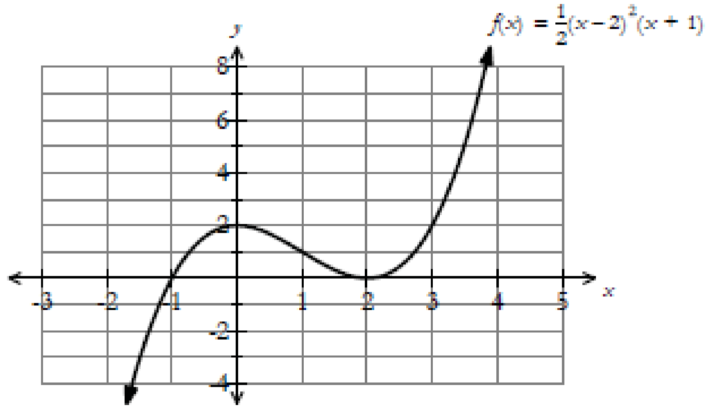


(ii)



(iii)





- Where is $f(x)$ increasing?
- What do you notice about the slopes of the tangents when $f(x)$ is increasing?
- Where is $f(x)$ decreasing?
- What do you notice about the slopes of the tangents when $f(x)$ is decreasing?
- Where is the slope of the tangent equal to zero?

Students should observe that on intervals where the tangent lines have positive slope, the function is increasing while on intervals where the tangent lines have negative slope the function is decreasing. The tangent lines have a zero slope at $x = 0$ and $x = 2$, which is where the graph changes direction.

Provide students the generalization for any function $f(x)$ that has a derivative on an interval:

- If $f\,'(x) > 0$ for all x in an interval, then $f(x)$ is increasing on that interval.
- If $f\,'(x) < 0$ for all x in an interval, then $f(x)$ is decreasing on that interval.
- If $f'(c) = 0$, then the graph of $f(x)$ has a horizontal tangent at $x = c$.

The previous functions were answered graphically. How do we determine intervals of increase/decrease algebraically?

Consider the function: $f(x) = x^3 - \frac{3}{2}x^2$.

Algebraically determine the intervals of increase/decrease. Determine where the local maximum and minimum exist.

1. $f(x) = 2x^3 - 3x^2$

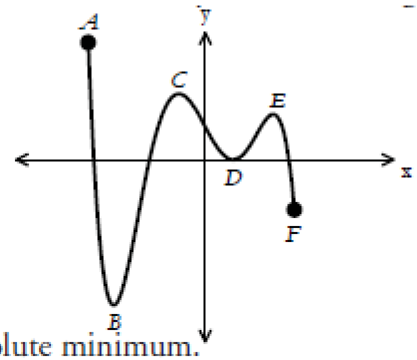
2. $f(x) = 3x^4 - 6x^2$

3. $f(x) = x^3 - 3x^2 - 9x + 5$

4. $f(x) = x^2 - 6x + 1$

graph with a focus on the following:

- Point A is an absolute maxiumum.
- Point B is both a local and absolute minimum.
- Point C and Point E are local maximum values.
- Point D is a local minimum.
- Point F is neither a local nor an absolute minimum.



1. Find the absolute extrema for $y = f(x)$ on the indicated interval.

a. $f(x) = 2x^3 - 3x^2 - 36x + 62 \quad [-3, 4]$

b. $f(x) = (x-1)^{\frac{1}{3}} \quad [-2, 2]$

2. Find the local extrema for $y = f(x)$.

a. $f(x) = x^3 - 3x$

b. $f(x) = x^4 - 8x^2 - 10$

3. Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 5$

a. Determine the critical numbers of $f(x)$.

b. Find the local extrema for $f(x)$.

c. Find the absolute extrema for $f(x)$ on $[-2, 4]$