Math 3200	Logarithms Recap to Date	Name: Key

1. Evaluate w/o a calculator

a. $\log_5 125 = 3$

b. $\log_3 \sqrt{27} = \frac{3}{2}$

c. $\log 10 = 1$

d. $\log_{25} \frac{1}{5} = \frac{-1}{2}$

$25^x = \frac{1}{5}$ $\quad 2x = -1$

$5^{2x} = 5^{-1}$

e. $\log_3 \sqrt{3} = x$

$\frac{1}{2} = x$

f. $\frac{1}{2}\log_2 16$

$\log_2 4 = 2$

2. Solve for x

a. $\log_6 x = 2$

$6^2 = x$

$36 = x$

b. $\log_5 21 = x$

$5^x = 21$

$x \approx 1.9$

c. $\log_x 8 = \frac{3}{2}$

$(x^{\frac{3}{2}})^{\frac{2}{3}} = (8)^{\frac{2}{3}}$

$x = 4$

d. $\log_4 x = -1$

$4^{-1} = x$

$\frac{1}{4} = x$

e. $\log_5 50 = x$

$5^x = 50$

$x \approx 2.4$

f. $\log_5 x = -2$

$5^{-2} = x$

$\frac{1}{25} = x$

3. Solve for x (calculator)

a. $2^x = 50$

$\log 2^x = \log 50$

$\frac{x \log 2}{\log 2} = \frac{\log 50}{\log 2}$

$x = 5.6$

b. $\frac{7}{7}(4)^x = \frac{21}{7}$

$4^x = 3$

$\log_4 3 = x$

$0.79 = x$

c. $4^x = 10$

$\log_4 10 = x$

$1.66 = x$

d. $5^{3x-1} = 50^x$

$\frac{(3x-1)\log 5}{\log 5} = \frac{x \log 50}{\log 5}$

$3x - 1 = x(2.43)$

$3x - 2.43x = 1$

$0.57x = 1$

$x = 1.75$

e. $4^{3x-1} = 10^{x-1}$

$\frac{(3x-1)\log 4}{\log 4} = \frac{(x-1)\log 10}{\log 4}$

$3x - 1 = (x-1)1.66$

$3x - 1 = 1.66x - 1.66$

$1.34x = -0.66$

$x = -0.49$

f. $\frac{(3)5^{2x+1}}{3} = \frac{45}{3}$

$5^{2x+1} = 15$

$\log_5 15 = 2x + 1$

$1.68 = 2x + 1$

$0.68 = 2x$

$0.34 = x$

4. Evaluate

a. $\log_5 30 = 2.11$

b. $\log_3 35 = 3.24$

c. $\log_6 100 = 2.57$

ex $\log_2 8 \cdot 4^2$

$\log_2 8.16$

$\log_2 8 + \log_2 4^2$

$3 + 2 \cdot (2)$

$= 7$

5. Solve for x

a. $\log_6 x = \log_3 27 - \log\ 10$

$\log_6 x = 3 - 1$

$\log_6 x = 2$

$6^2 = x$

$36 = x$

b. $x = 3\log_{\frac{1}{2}} 8\sqrt{32}$

$x = 3\log_{\frac{1}{2}}\left(2^3 \cdot 2^{\frac{5}{2}}\right)$

$x = 3\log_{\frac{1}{2}}\left(2^{\frac{11}{2}}\right)$

$x = 3 \cdot \left(\frac{11}{2}\right)\log_{\frac{1}{2}} 2$

$x = \frac{33}{2} \cdot (-1)$

$x = -\frac{33}{2}$

c. $x = \log_6 36 - \log_4 8 + 5\log_{\frac{1}{3}} \sqrt{27}$

$x = 2 - \left(\frac{3}{2}\right) + 5\left(-\frac{3}{2}\right)$

$x = 2 - \frac{3}{2} - \frac{15}{2}$

$x = -7$

d. $\log_x 4 = \log\frac{1}{10} + \log_3 3 + \log_8 2$

$\log_x 4 = (-1) + 1 + \frac{1}{3}$

$\log_x 4 = \frac{1}{3}$

$x^{\frac{1}{3}\cdot 3} = 4^3$

$x = 64$

e. $2\log_3(x+3) - \log_3(x+1) = 2$

$\log_3 (x+3)^2 - \log_3 (x+1) = 2$

$\log_3 \frac{(x+3)^2}{(x+1)} = 2$

$9 = \frac{x^2+6x+9}{x+1}$

$9x+9 = x^2+6x+9$

$0 = x^2 - 3x$

$0 = x(x-3)$

$x = 0 \quad x = 3$

f. $\log_2(x+2) + \frac{1}{3}\log_2(8x^3) = 16^{\frac{1}{2}}$

$\log_2(x+2)\cdot(8x^3)^{\frac{1}{3}} = 4$

$\log_2(x+2)\cdot 2x) = 4$

$16 = 2x^2 + 4x$

$0 = 2x^2 + 4x - 16$

$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

$x = -4 \quad x = 2$

g. $\log(x-2)^2 - \log(x+3.6) = 1$

$\log\left(\frac{x^2-4x+4}{x+3.6}\right) = 1$

$10 = \frac{x^2-4x+4}{x+3.6}$

$10x + 36 = x^2 - 4x + 4$

$0 = x^2 - 14x - 32$

$0 = (x-16)(x+2)$

$x = 16 \quad x = -2$

h. $\log_7(x-2) + \log_7(x+4) = 1$

$\log_7(x-2)(x+4) = 1$

$7 = x^2 + 2x - 8$

$0 = x^2 + 2x - 15$

$0 = (x+5)(x-3)$

$x = -5 \quad x = 3$

✳

6. Two ATVs were bought at the same time. One ATV cost $10 000 and depreciates at 22% yearly. The other ATV cost $8000 and depreciates at 18% yearly. Algebraically determine the number of years it will take for the two ATVs to be of equal value.

$A = 10\,000(0.78)^t$ $\quad A = 8000(0.82)^t$

$\frac{10000(0.78)^t}{8000} = \frac{8000(.82)^t}{8000}$

$1.25(0.78)^t = \log(.82)^t$

$\log 1.25 + t\log(.78) = t\log(.82)$

$\log 1.25 = t\log(.82) - t\log(.78)$

$\frac{\log 1.25}{\log(.82)-\log(.78)} = \frac{t(\log(.82)-\log(.78))}{\log(.82)-\log(.78)}$

$4.46 = t$

about 4.5 years

7. a. The half-life of a certain element is 60 years. If the initial amount is 850 grams, how long will it take to reduce to 300 grams?

$\frac{300}{850} = \frac{850(\frac{1}{2})^{\frac{t}{60}}}{850}$

$0.3529 = (\frac{1}{2})^{\frac{t}{60}}$

$\frac{\log(0.3529)}{\log(\frac{1}{2})} = \frac{\frac{t}{60}\log(\frac{1}{2})}{\log(\frac{1}{2})}$

$1.5 = \frac{t}{60}$

$90 = t$

90 years

b. A new Nissan Titan is purchased for $48000. If it depreciates by 25% every 3 years, how long will it take to be worth $23000?

$\frac{23\,000}{48000} = \frac{48000(0.75)^{\frac{t}{3}}}{48000}$

$0.479 = (0.75)^{\frac{t}{3}}$

$\frac{\log(0.479)}{\log(0.75)} = \frac{\frac{t}{3}\log(0.75)}{\log(0.75)}$

$2.559 = \frac{t}{3}$

$7.677 = t$

about 7.7 years

c. Anna earns $5 working for Mr. LeDrew as a hall monitor in the morning. If her contract states that her salary will increase by 20% every 5 days, how long will it take for her original salary will double?

$2 = (1.20)^{\frac{t}{5}}$

$\frac{\log 2}{\log(1.20)} = \frac{\frac{t}{5}\log(1.20)}{\log(1.20)}$

$3.8 = \frac{t}{5}$

$19 \text{ days} = t$

d. Noah invests his soccer bonus $5000into a fund that earns 15% interest every 2 years. How long will it take for his money to be worth $12,000?

$\frac{12\,000}{5000} = \frac{5000(1+0.15)^{\frac{t}{2}}}{5000}$

$2.4 = (1.15)^{\frac{t}{2}}$

$\frac{\log 2.4}{\log(1.15)} = \frac{\frac{t}{2}\log(1.15)}{\log(1.15)}$

$6.26 = \frac{t}{2}$

$12.5 \text{ years} = t$

e. A new car was purchased for \$32 000 and sold 4 years later for \$12 500. If the car continues to depreciate at the same rate, how long, to the nearest tenth of a year, will it take for the car to depreciate from \$12 500 to \$5400.

$\frac{12500}{32000} = \frac{32000}{32000}(1-r)^4$

$0.3906 = (1-r)^4$

$\frac{\log(0.3906)}{4} = \frac{4}{4}\log(1-r)$

$-0.102 = \log(1-r)$

$10^{-0.102} = 1-r$

$r = 1 - 0.79$

$r = 0.21$

8. * Algebraically solve: $2(5)^{2x} - 9(5)^x - 5 = 0$

let $y = 5^x$

$2y^2 - 9y - 5 = 0$

$(2y+1)(y-5) = 0$

$y = -\frac{1}{2}$ $y = 5$

$5^x = -\frac{1}{2}$ $5^x = 5$

No solution $x = 1$

$\frac{5400}{12500} = \frac{12500}{12500}(0.79)^t$

$0.432 = (0.79)^t$

$\log(0.432) = t\log(0.79)$

$3.6 \text{ years} = t$

9. Write $\frac{1}{2}\log 7 + \frac{3}{2}\log x - \frac{1}{2}\log y$ as a single logarithm.

$= \log 7^{\frac{1}{2}} + \log x^{3/2} - \log y^{\frac{1}{2}}$

$= \log \frac{\sqrt{7}\cdot\sqrt{x^3}}{\sqrt{y}}$ or $= \log\sqrt{\frac{7x^3}{y}}$

10. For the function $y = \frac{1}{3}\log_2(3x+6)+4$: $y = \frac{1}{3}\log_2 3(x+2)+4$

a. What is the Domain?

$\{x \mid x > -2, x \in R\}$

b. Write a mapping rule.

$(x,y) \to (\frac{1}{3}x - 2, \frac{1}{3}y + 4)$

c. What is the y-intercept?

$y = \frac{1}{3}\log_2(0+6)+4$

$y = \frac{1}{3}\log_2 6 + 4$ $(0, 4.9)$

d. What is the x-intercept?

$0 = \frac{1}{3}\log_2(3x+6)+4$

$-4 = \frac{1}{3}\log_2(3x+6)$

$-12 = \log_2(3x+6)$

$2^{-12} = 3x+6$

$-2 = x$ $(-2, 0)$

e. What is the equation of the VA?

$x = -2$

1. What is the exact value of x: $7 = 2^{x+1}$

A) $\log\left(\frac{7}{2}\right) - 1$ B) $\frac{\log 7}{\log 2} + 1$

C) $\log\left(\frac{7}{2}\right) + 1$ D) $\frac{\log 7}{\log 2} - 1$

$\log 7 = (x+1)\log 2$

$x+1 = \frac{\log 7}{\log 2}$

2. Solve for x: $2^{3x-1} = 8^{2x+1}$

A) $x = -\frac{4}{3}$ B) $x = -1$

C) $x = -\frac{2}{3}$ D) $x = -\frac{3}{4}$

$(3x-1)\log 2 = (2x+1)\log 8$

$3x\log 2 - 2x\log 8 = \log 8 + \log 2$

$x = \frac{\log 8 + \log 2}{3\log 2 - 2\log 8}$

$= -4/3$

3. Express $\log\left(\frac{x^2}{10y^3}\right)$ in terms of $\log x$ and $\log y$.

A) $2\log x - 1 - 3\log y$

B) $2\log x - 1 + 3\log y$

C) $2\log x - 10 - 3\log y$

D) $2\log x - 10 + 3\log y$

$2\log x - [\log 10 + 3\log y]$

$2\log x - 1 - 3\log y$

4. What is the value of $\log_3 \sqrt{27}$?

A) $x = \frac{2}{9}$ B) $x = \frac{2}{3}$

C) $x = \frac{3}{2}$ D) $x = \frac{9}{2}$

$3^x = \sqrt{27}$

$3^x = 3^{3/2}$

$x = 3/2$

5. What is the x-intercept of $y = \log_2(x+7)$?

A) -7 B) -6

C) 0 D) 3

$0 = \log_2(x+7)$

$2^0 = x + 7$

$1 = x + 7$

$x = -6$

7. What is the value of x: $3\left(5^{\frac{x}{2}}\right)=12$?

A) $\frac{\log 5}{2\log 4}$ B) $\frac{2\log 5}{\log 4}$

C) $\frac{\log 4}{2\log 5}$ D) $\frac{2\log 4}{\log 5}$

$5^{\frac{x}{2}} = 4$

$\frac{x}{2}\log 5 = \log 4$

$\frac{x}{2} = \frac{\log 4}{\log 5}$

$x = \frac{2\log 4}{\log 5}$

8. Solve for x: $(2x-1)^{\frac{1}{2}}=\sqrt{3}$.

A) 0.74 B) 1.37

C) 2 D) 2.73

$2x-1 = 3$

$2x = 4$

$x = 2$

9. What is the inverse of $y=3^x$?

A) $y=\log_x 3$ B) $y=\log_3 x$

C) $x=\log_3 y$ D) $x=\log_y 3$

$x = 3^y$

$\log_3 x = y$

10. Solve for x: $\log_5(5x+2)=\frac{1}{2}\log_5 49+\log_5 16$.

A) 4.2 B) 22

C) 78 D) 156.4

$\log_5(5x+2) = \log_5(7)(16)$

$5x+2 = 112$

$5x = 110$

$x = 22$

11. Which is $m\log_p n = q$ written in exponential form?

A) $p^m = n^q$ B) $p^q = n^m$

C) $p^q = mn$ D) $p^{qm} = n$

$\log_p n^m = q$

$p^q = n^m$

12. Solve for x: $\log_7(2x)+\log_7(x-3)=\log_7 8$

A) $x=1$ B) $x=4$

C) $x=5$ D) $x=6$

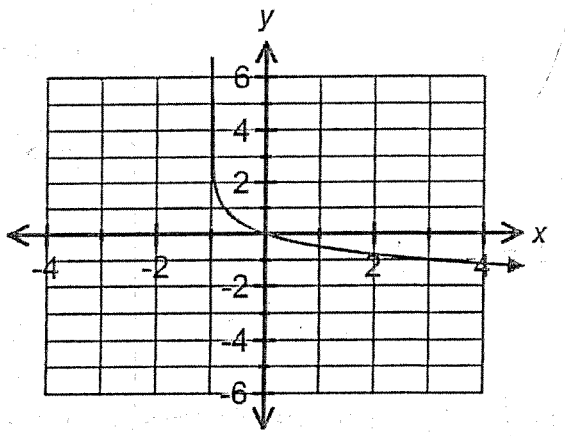
$(2x)(x-3) = 8$

$2x^2-6x = 8$

$2x^2-6x-8=0$

$2(x^2-3x-4)=0$

14. Which function best represents the graph shown below?



~~A) $y = -\log_4(x-1)$~~

B) $y = -\log_4(x+1)$

~~C) $y = \log_4(x-1)$~~

D) $y = \log_4(x+1)$

VA $x = -1$

15. Which expression is equivalent to $\log\frac{\sqrt{B}A}{C^4}$?

$\frac{1}{2}\log B + \log A - 4\log C$

A) $\frac{1}{2}\log B + \log A - 4\log C$

B) $\frac{1}{2}\log B + 4\log\frac{A}{C}$

C) $\frac{\frac{1}{2}\log(B+A)}{4\log C}$

D) $\frac{\frac{1}{2}\log B + \log A}{4\log C}$

16. What is the domain of $y = \log_3(5-x)$?

$(-x+5)$

A) $x > 5, x\varepsilon R$

B) $x > -5, x\varepsilon R$

$-x = -5$

$x = 5$

$x \leq 5$

C) $x < 5, x\varepsilon R$

$< -5, x\varepsilon R$

17. What is the domain of $y = 3\log_2(-2(x-4)) + 7$?

A) $x > 4, x\varepsilon R$

$-2(x-4) = 0$ $x = 4$

B) $x < 4, x\varepsilon R$

C) $x > -8, x\varepsilon R$

#19a) $\log_{15}(3-x) + \log_{15}(1-x) = 1$

$\log_{15}(3-x)(1-x) = 1$

$15^1 = 3 - 4x + x^2$

$0 = x^2 - 4x - 12$

$0 = (x-6)(x+2)$

~~$x = 6$~~ or $x = -2$

b) $\log_9(x^2 - 2x - 15) = 1$

$9^1 = x^2 - 2x - 15$

$0 = x^2 - 2x - 24$

$0 = $ ~~$(x-12)(x+2)$~~

~~$x = 12$ or $x = -2$~~

$0 = (x-6)(x+4)$

$x = 6$ or ~~$x = -4$~~

c) $2\log_3 x - \log_3(2x+3) = 0$

$\log_3\left(\frac{x^2}{2x+3}\right) = 0$

$3^0 = \frac{x^2}{2x+3}$

$2x + 3 = x^2$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3$ or ~~$x = -1$~~

d) $4^1 = x^2 - 3x$

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$x = 4$ or $x = -1$

e) $\log_4(\log_3 x) = 0$

$4^0 = \log_3 x$

$1 = \log_3 x$

$3^1 = x$

f) $\log_3(\log_5 125) = 1$

$3^1 = \log_5 125$

$x^3 = 125$

$x = 125^{1/3}$

$x = 5$

(g) $6^{\log x} = \frac{1}{36}$

$6^{\log x} = 6^{-2}$

$\log x = -2$

$10^{-2} = x$

$x = \frac{1}{100}$

(h) $\log_x 8 = \frac{3}{4}$

$x^{3/4} = 8$

$x = 8^{4/3}$

$x = 16$

#20

i) $\log(x^2+12) = \log 8x$

$x^2 + 12 = 8x$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 6$ or $x = 2$

#20. $-5 = \log_c \frac{1}{32}$

$c^{-5} = \frac{1}{32}$

$c^{-5(-1/5)} = \left(\frac{1}{32}\right)^{-1/5}$

$\boxed{c = 2}$

$(\overset{x}{K}, \overset{y}{256}) \rightarrow$ inverse

$(\overset{x}{256}, \overset{y}{K})$

$K = \log_2 256$

$2^K = 256$

$2^K = 2^8$

$K = 8$

#21 $20 = 50\left(\frac{1}{2}\right)^{x/25}$

$\frac{2}{5} = \left(\frac{1}{2}\right)^{x/25}$

$\log\left(\frac{2}{5}\right) = \frac{x}{25}\log\left(\frac{1}{2}\right)$

$\frac{x}{25} = \frac{\log\left(\frac{2}{5}\right)}{\log\left(\frac{1}{2}\right)}$

$x = 25\,\frac{\log(2/5)}{\log(1/2)}$

$x = 33$

#22. $34000(0.25)^{x/6} = 18000(0.5)^{x/4}$

$\log\left[34000(0.25)^{x/6}\right] = \log\left[18000(0.5)^{x/4}\right]$

$\log 34000 + \frac{x}{6}\log 0.25 = \log 18000 + \frac{x}{4}\log 0.5$

$\frac{x}{6}\log 0.25 - \frac{x}{4}\log 0.5 = \log 18000 - \log 34000$

$x\left[\frac{1}{6}\log 0.25 - \frac{1}{4}\log 0.5\right] = \log 18000 - \log 34000$

$x = \frac{\log 18000 - \log 34000}{\frac{1}{6}\log 0.25 - \frac{1}{4}\log 0.5}$

$x = \frac{-0.27621}{-0.02509}$

$\doteq$ 11 years

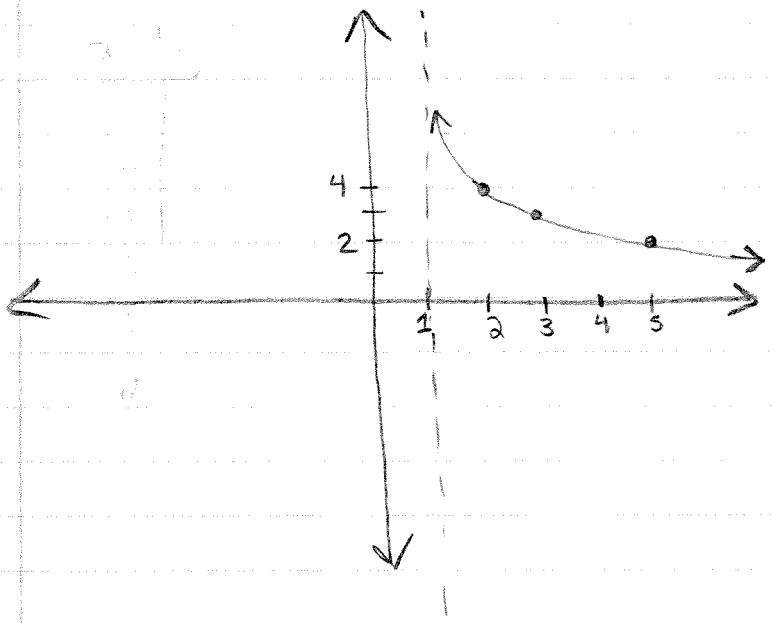
23. $y = -\log_2(x-1)+4$

*Reflection in x-axis

VT = 4

HT = 1

mapping rule: $(x,y) \to (x+1, -y+4)$



x-int.

$0 = -\log_2(x-1)+4$

$-4 = -\log_2(x-1)$

$2^4 = x-1$

$16 = x-1$ (17, 0)

$17 = x$