Extra Practice 6.2

a) $\sin 75^\circ \cos 15^\circ + \cos 75^\circ \sin 15^\circ$

$= \sin(75^\circ + 15^\circ)$

$= \sin 90^\circ$

$= 1$

b) $\cos \frac{5\pi}{12} \cos \frac{\pi}{12} - \sin \frac{5\pi}{12} \sin \frac{\pi}{12}$

$= \cos \left[\frac{5\pi}{12} + \frac{\pi}{12}\right]$

$= \cos \left[\frac{6\pi}{12}\right]$

$= \cos \left[\frac{\pi}{2}\right] = 0$

c) $\cos 75^\circ = \cos[45^\circ + 30^\circ]$

$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

d) $\overset{165^\circ}{\sin \frac{11\pi}{12}} = \sin\left[\overset{135^\circ}{\frac{3\pi}{4}} + \overset{30^\circ}{\frac{\pi}{6}}\right]$

$= \sin \frac{3\pi}{4} \cos \frac{\pi}{6} + \cos \frac{3\pi}{4} \sin \frac{\pi}{6}$

$= \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + -\frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

(e) $\dfrac{\tan \frac{2\pi}{3} + \tan \frac{\pi}{12}}{1 - \tan \frac{2\pi}{3} \tan \frac{\pi}{12}} = \tan\left(\frac{2\pi}{3} + \frac{\pi}{12}\right)$

$= \tan \frac{3\pi}{4}$

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f) $\cot 165° = \dfrac{1}{\tan 165°}$

$= \dfrac{1}{\tan[135° + 30°]}$

$= \dfrac{1}{\dfrac{\tan 135 + \tan 30}{1 - \tan 135 \tan 30}}$

$= \dfrac{1 - \tan 135 \tan 30}{\tan 135 + \tan 30}$

$= \dfrac{1 - (-1)\left(\frac{\sqrt{3}}{3}\right)}{-1 + \frac{\sqrt{3}}{3}}$

$= \dfrac{\frac{3 + \sqrt{3}}{3}}{\frac{-3 + \sqrt{3}}{3}}$

$= \dfrac{3 + \sqrt{3}}{-3 + \sqrt{3}} \cdot \dfrac{(-3 - \sqrt{3})}{(3 - \sqrt{3})}$

$= \dfrac{-9 - 3\sqrt{3} - 3\sqrt{3} - 3}{9 - 3}$

$= \dfrac{-12 - 6\sqrt{3}}{6}$

$= -2 - \sqrt{3}$

g) $\cos(-165°)$

$= \cos(195°) = \cos(135° + 60°)$

$= \cos 135 \sin 60 - \sin 135 \cos 60$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \dfrac{-\sqrt{6} - \sqrt{2}}{4}$

#2a) $\cos 2x \cos x + \sin 2x \sin x \quad = \cos(2x - x) = \cos x$

$= \cos 2x \cos x + 2 \sin x \cos x \sin x$

$= \cos 2x \cos x + 2\sin^2 x \cos x$

$= (1 - 2\sin^2 x) \cos x + 2\sin^2 x \cos x$

$= \cos x - \cancel{2\sin^2 x \cos x} + \cancel{2\sin^2 x \cos x}$

$= \cos x$

#2b) $\sin(30° + x) + \sin(30° - x)$

$= \sin 30 \cos x + \cancel{\cos 30 \sin x} + \sin 30 \cos x - \cancel{\cos 30 \sin x}$

$= 2 \sin 30 \cos x$

$= 2 \cdot \frac{1}{2} \cos x$

$= \cos x$

2\. c) $= \cancel{\cos x} \cos y \cdot \frac{\sin x}{\cancel{\cos x}} + \cos x \cancel{\cos y} \cdot \frac{\sin y}{\cancel{\cos y}}$

$= \cos y \cdot \sin x + \cos x \sin y$

$= \sin(x+y)$

d) $= \cos 2(10°)$

$= \cos 20°$

e) $= \sin 2(35°)$

$= \sin 70°$

f) $= \tan 2(25°)$

$= \tan 50°$

3\. a) $LHS = \cos\frac{\pi}{2}\cos\theta - \sin\frac{\pi}{2}\sin\theta$

$= (0)\cos\theta - (1)\sin\theta$

$= -\sin\theta$

$\therefore LHS = RHS$

b) $LHS = \sin\pi\cos x + \cos\pi\sin x$

$= (0)\cos x + (-1)\sin x$

$= -\sin x$

$\therefore LHS = RHS$

4. Simplify.

a) $\sin(30^\circ + x) + \sin(30^\circ - x)$

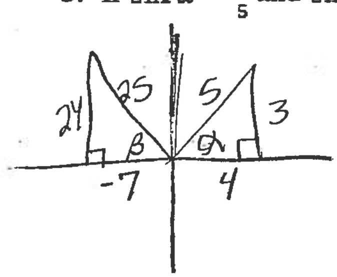
b) $\cos\left(\frac{3\pi}{2} + \theta\right) + \cos\left(\frac{3\pi}{2} - \theta\right)$

c) $\frac{1+\cos 2x}{\cot x}$

d) $(1 - \sin^2 x)(1 - \tan^2 x)$

e) $\sin x \tan x + \cos 2x \sec x$

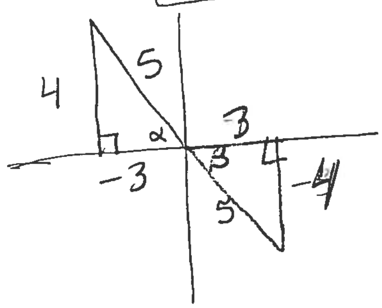
5. If $\sin\alpha = \frac{3}{5}$ and $\sin\beta = \frac{24}{25}$, and if $0 < \alpha < \frac{\pi}{2} < \beta < \pi$, find $\sin(\alpha+\beta)$.



$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\frac{3}{5}\right)\left(\frac{-7}{25}\right) + \left(\frac{4}{5}\right)\left(\frac{24}{25}\right)$$
$$= \frac{-21}{125} + \frac{96}{125}$$
$$= \frac{75}{125} = \boxed{\frac{3}{5}}$$

change

6. If $\boxed{\frac{\pi}{2} < \alpha < \beta < \pi,}$ and $\sin\alpha = \frac{4}{5}$ and $\tan\beta = \cancel{\frac{3}{4}}$ $-\frac{4}{3}$, find $\cos(\alpha+\beta)$ and $\tan(\alpha+\beta)$.



$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \left(\frac{-3}{5}\right)\left(\frac{4}{5}\right) - \left(\frac{4}{5}\right)\left(\frac{-3}{5}\right)$$
$$= \frac{-12}{25} + \frac{12}{25}$$
$$\cos(\alpha+\beta) = 0$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\left(\frac{4}{-3}\right) + \left(\frac{-4}{3}\right)}{1 - \left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right)} = \frac{-\frac{8}{3}}{1 - \frac{16}{9}} = \frac{-\frac{8}{3}}{-\frac{7}{9}}$$
$$= -\frac{8}{3} \cdot \frac{9}{-7} = \boxed{\frac{24}{7}}$$

4. a) $= \sin 30^\circ \cos x + \cos 30^\circ \sin x + \sin 30^\circ \cos x - \cancel{\cos 30^\circ \sin x}$

$= \frac{1}{2}\cos x + \frac{1}{2}\cos x$

$= \boxed{\cos x}$

b) $= \cos\frac{3\pi}{2}\cos\theta - \sin\frac{3\pi}{2}\sin\theta$

$= (0\cos\theta) - (-1)\sin\theta$

$= \boxed{\sin\theta}$

c) $= \dfrac{1+\cos^2 x - \sin^2 x}{\cot x}$

$= \dfrac{\cos^2 x + \cos^2 x}{\cot x}$

$= \dfrac{2\cos^2 x}{\cot x}$

$= 2\cos^{\cancel{2}} x \cdot \dfrac{\sin x}{\cancel{\cos x}} = 2\cos x \sin x$

$= \sin 2x$

d) $= \cos^2 x(1-\tan^2 x)$

$= \cos^2 x - \cos^2 x \cdot \tan^2 x$

$= \cos^2 x - \cancel{\cos^2 x} \cdot \dfrac{\sin^2 x}{\cancel{\cos^2 x}}$

$= \cos^2 x - \sin^2 x$

$= \boxed{\cos 2x}$

e) $= \sin x \tan x + (\cos^2 x - \sin^2 x) \cdot \dfrac{1}{\cos x}$

$= \sin x \tan x + \cos^{\cancel{2}} x \cdot \dfrac{1}{\cancel{\cos x}} - \dfrac{\sin^2 x}{\cos x}$

$= \cancel{\sin x \tan x} + \cos x - \cancel{\tan x \sin x}$

$= \boxed{\cos x}$