

Extra Practice 6.2

$$\begin{aligned} \text{a)} \quad \sin 75^\circ \cos 15^\circ + \cos 75^\circ \sin 15^\circ &= \sin(75^\circ + 15^\circ) \\ &= \sin 90^\circ \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad \cos \frac{5\pi}{12} \cos \frac{\pi}{12} - \sin \frac{5\pi}{12} \sin \frac{\pi}{12} &= \cos \left[\frac{5\pi}{12} + \frac{\pi}{12} \right] \\ &= \cos \left[\frac{6\pi}{12} \right] \\ &= \cos \left[\frac{\pi}{2} \right] = 0 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad \cos 75^\circ &= \cos [45^\circ + 30^\circ] \\ &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \text{d)} \quad \sin \frac{11\pi}{12} &= \sin \left[\frac{3\pi}{4} + \frac{\pi}{6} \right] \\ &= \sin \frac{3\pi}{4} \cos \frac{\pi}{6} + \cos \frac{3\pi}{4} \sin \frac{\pi}{6} \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{2}}{2} \right) \cdot \frac{1}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \text{e)} \quad \frac{\tan \frac{2\pi}{3} + \tan \frac{\pi}{12}}{1 - \tan \frac{2\pi}{3} \tan \frac{\pi}{12}} &= \tan \left(\frac{2\pi}{3} + \frac{\pi}{12} \right) \\ &= \tan \frac{3\pi}{4} \end{aligned}$$

$$f) \cot 165^\circ = \frac{1}{\tan 165^\circ}$$

$$= \frac{1}{\tan [135^\circ + 30^\circ]}$$

$$= \frac{1}{\frac{\tan 135^\circ + \tan 30^\circ}{1 - \tan 135^\circ \tan 30^\circ}}$$

$$= \frac{1 - \tan 135^\circ \tan 30^\circ}{\tan 135^\circ + \tan 30^\circ}$$

$$= \frac{1 - (-1) \left(\frac{\sqrt{3}}{3}\right)}{-1 + \frac{\sqrt{3}}{3}}$$

$$= \frac{3 + \sqrt{3}}{3}$$

$$\frac{-3 + \sqrt{3}}{3}$$

$$= \frac{3 + \sqrt{3}}{-3 + \sqrt{3}} \cdot \frac{(3 - \sqrt{3})}{(3 - \sqrt{3})}$$

$$= \frac{-9 - 3\sqrt{3} - 3\sqrt{3} - 3}{9 - 3}$$

$$= \frac{-12 - 6\sqrt{3}}{6}$$

$$= -2 - \sqrt{3}$$

- 3.13

$$g) \cos(-165^\circ) = \cos(195^\circ) = \cos(135^\circ + 60^\circ)$$

$$= \cos 135^\circ \sin 60^\circ - \sin 135^\circ \cos 60^\circ$$

$$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= -\frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\#2a) \cos 2x \cos x + \sin 2x \sin x = \cos(2x - x)$$

$$= \cos 2x \cos x + 2 \sin x \cos x \sin x = \cos x$$

$$= \cos 2x \cos x + 2 \sin^2 x \cos x$$

$$= (1 - 2 \sin^2 x) \cos x + 2 \sin^2 x \cos x$$

$$= \cos x - 2 \sin^2 x \cos x + 2 \sin^2 x \cos x$$

$$= \cos x$$

$$\#2b) \sin(30^\circ + x) + \sin(30^\circ - x)$$

$$= \sin 30^\circ \cos x + \cos 30^\circ \sin x + \sin 30^\circ \cos x - \cos 30^\circ \sin x$$

$$= 2 \sin 30^\circ \cos x$$

$$= 2 \cdot \frac{1}{2} \cos x$$

$$= \cos x$$

$$\begin{aligned}
 2. c) &= \frac{\cos x \cos y \cdot \sin x}{\cos x} + \frac{\cos x \cos y \cdot \sin y}{\cos y} \\
 &= \cos y \cdot \sin x + \cos x \sin y \\
 &= \sin(x+y)
 \end{aligned}$$

$$\begin{aligned}
 d) &= \cos 2(10^\circ) \\
 &= \cos 20^\circ
 \end{aligned}$$

$$\begin{aligned}
 e) &= \sin 2(35^\circ) \\
 &= \sin 70^\circ
 \end{aligned}$$

$$\begin{aligned}
 f) &= \tan 2(25^\circ) \\
 &= \tan 50^\circ
 \end{aligned}$$

$$\begin{aligned}
 3. a) \text{ LHS} &= \cos \frac{\pi}{2} \cos \theta - \sin \frac{\pi}{2} \sin \theta \\
 &= (0) \cos \theta - (1) \sin \theta \\
 &= -\sin \theta \\
 &\therefore \text{LHS} = \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 b) \text{ LHS} &= \sin \pi \cos x + \cos \pi \sin x \\
 &= (0) \cos x + (-1) \sin x \\
 &= -\sin x \\
 &\therefore \text{LHS} = \text{RHS}
 \end{aligned}$$

4. Simplify.

a) $\sin(30^\circ + x) + \sin(30^\circ - x)$

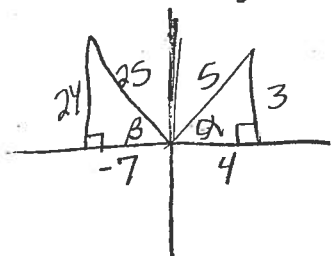
b) $\cos\left(\frac{3\pi}{2} + \theta\right) + \cos\left(\frac{3\pi}{2} - \theta\right)$

c) $\frac{1 + \cos 2x}{\cot x}$

d) $(1 - \sin^2 x)(1 - \tan^2 x)$

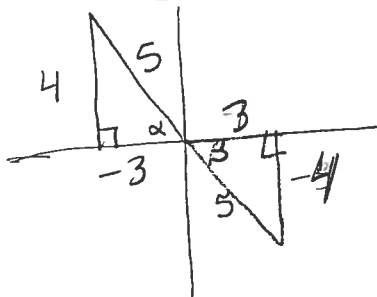
e) $\sin x \tan x + \cos 2x \sec x$

5. If $\sin \alpha = \frac{3}{5}$ and $\sin \beta = \frac{24}{25}$, and if $0 < \alpha < \frac{\pi}{2} < \beta < \pi$, find $\sin(\alpha + \beta)$.



$$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &= \left(\frac{3}{5}\right)\left(-\frac{7}{25}\right) + \left(\frac{4}{5}\right)\left(\frac{24}{25}\right) \\ &= \frac{-21}{125} + \frac{96}{125} \\ &= \frac{75}{125} = \boxed{\frac{3}{5}} \end{aligned}$$

6. If $\frac{\pi}{2} < \alpha < \beta < \pi$, and $\sin \alpha = \frac{4}{5}$ and $\tan \beta = \frac{-4}{3}$ find $\cos(\alpha + \beta)$ and $\tan(\alpha + \beta)$.



$$\begin{aligned} \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ &= \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) - \left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) \\ &= \frac{-12}{25} + \frac{12}{25} \end{aligned}$$

$$\cos(\alpha + \beta) = 0$$

$$\begin{aligned} \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\left(-\frac{4}{3}\right) + \left(-\frac{4}{3}\right)}{1 - \left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right)} = \frac{-\frac{8}{3}}{1 - \frac{16}{9}} = \frac{-\frac{8}{3}}{-\frac{7}{9}} \\ &= -\frac{8}{3} \cdot \frac{9}{-7} = \boxed{\frac{24}{7}} \end{aligned}$$

$$\begin{aligned}
 4. a) &= \sin 30^\circ \cos x + \cos 30^\circ \sin x + \sin 30^\circ \cos x - \cos 30^\circ \sin x \\
 &= \frac{1}{2} \cos x + \frac{1}{2} \cos x \\
 &= \boxed{\cos x}
 \end{aligned}$$

$$\begin{aligned}
 b) &= \cos \frac{3\pi}{2} \cos \theta - \sin \frac{3\pi}{2} \sin \theta \\
 &= (0 \cos \theta) - (-1) \sin \theta \\
 &= \boxed{\sin \theta}
 \end{aligned}$$

$$\begin{aligned}
 d) &= \cos^2 x (1 - \tan^2 x) \\
 &= \cos^2 x - \cos^2 x \cdot \tan^2 x \\
 &= \cos^2 x - \frac{\cos^2 x \cdot \sin^2 x}{\cos^2 x} \\
 &= \cos^2 x - \sin^2 x \\
 &= \boxed{\cos 2x}
 \end{aligned}$$

$$\begin{aligned}
 e) &= \sin x \tan x + (\cos^2 x - \sin^2 x) \cdot \frac{1}{\cos x} \\
 &= \sin x \tan x + \frac{\cos^2 x}{\cos x} - \frac{\sin^2 x}{\cos x} \\
 &= \sin x \tan x + \cos x - \frac{\sin^2 x}{\cos x} \\
 &= \boxed{\cos x}
 \end{aligned}$$

$$\begin{aligned}
 c) &= \frac{1 + \cos^2 x - \sin^2 x}{\cot x} \\
 &= \frac{\cos^2 x + \cos^2 x}{\cot x} \\
 &= \frac{2 \cos^2 x}{\cot x} \\
 &= 2 \cos^2 x \cdot \frac{\sin x}{\cos x} = 2 \cos x \sin x \\
 &= \sin 2x
 \end{aligned}$$